Holographic Renormalization Group Flows

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Outlook

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Holographic picture for deviation from conformality

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- **Holographic RGF for simple models**

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- **Holographic RGF for simple models**
- **Exact HRGF for two exp potential**
- **HRGF for chemical potential** $\mu \neq 0$
- **HRGF for anizotropic models**

Holographic picture for deviation from conformality

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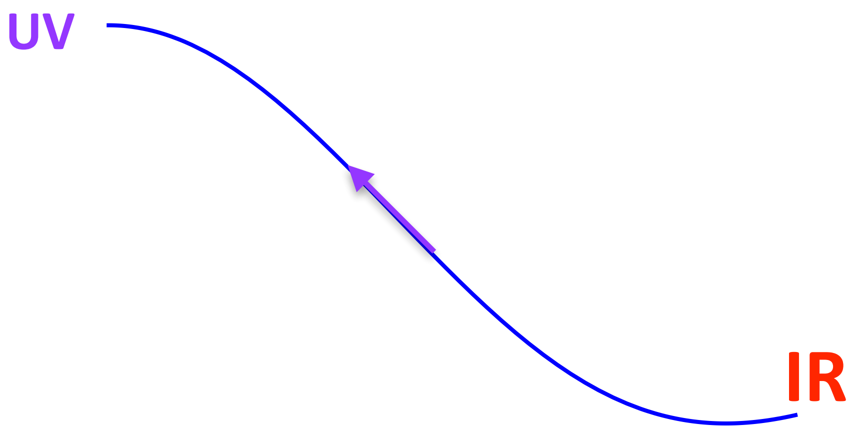

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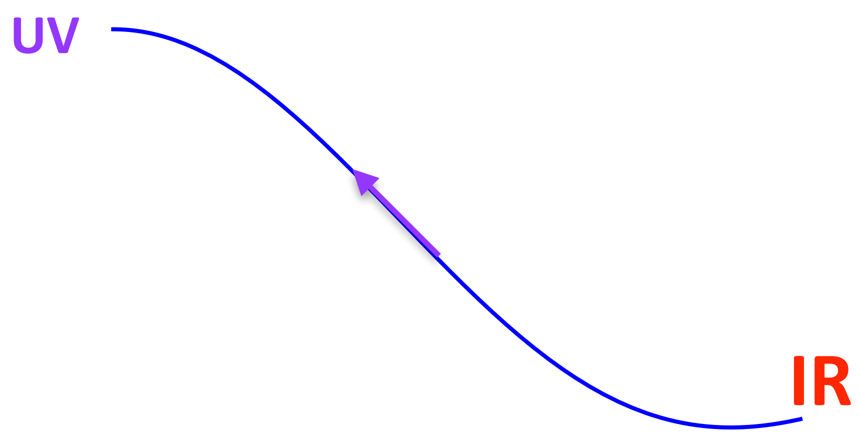

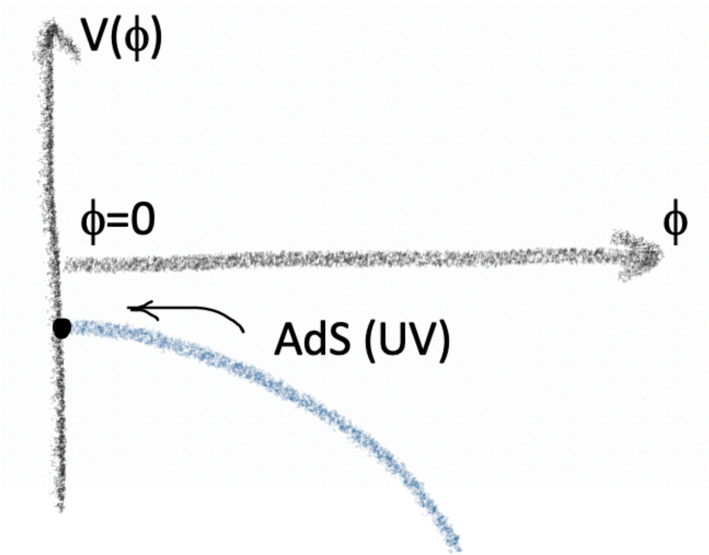

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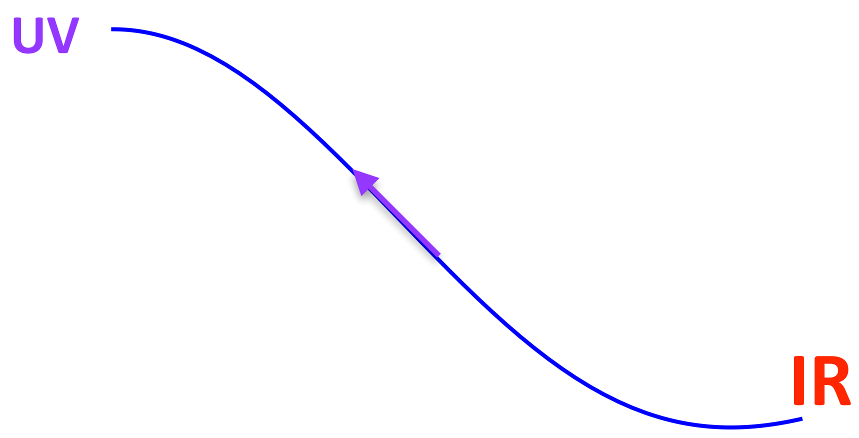

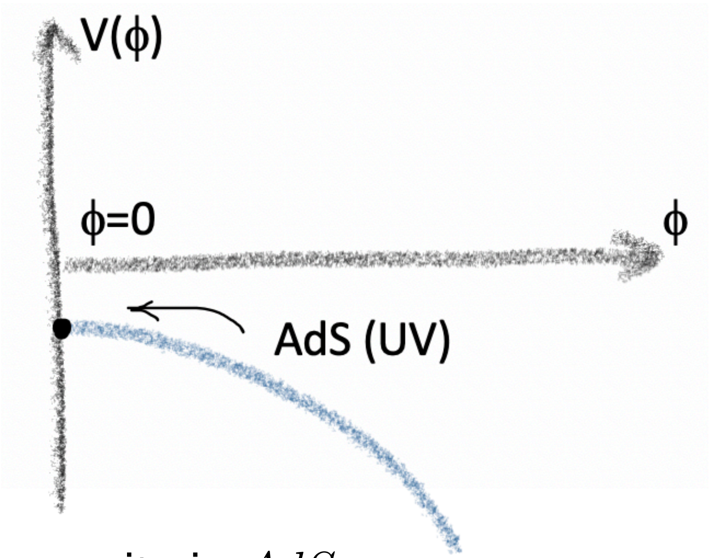

- d-dim CFT has a description in terms of $d+1$-dim gravity in AdS:
$S = \int dx^d du \sqrt{-g}(R - \Lambda)$.
- An operator $\mathcal{O}(x)$ corresponds to a dynamical bulk field $\phi(x,u)$
- $\phi(x,0)$ – a source for the $\mathcal{O}$ in the CFT

$$S = \int dx^d du \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right].$$

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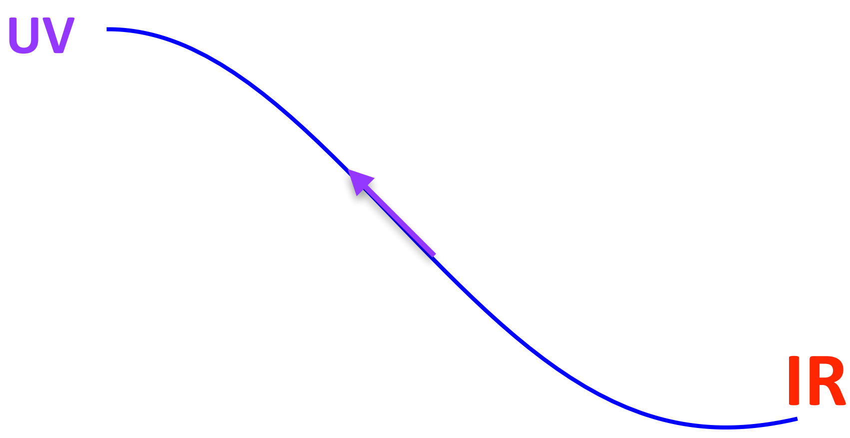

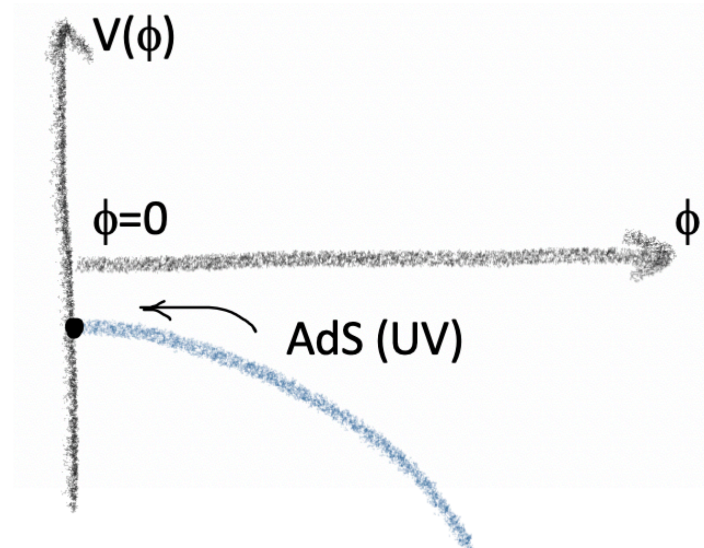

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- $\phi(x,u) = \alpha u^{d-\Delta} + \ldots \Leftrightarrow S = S_{CFT} + \int d^4x \alpha \mathcal{O}(x)$
- $\alpha = 0$ – undeformed CFT, bulk scalar – const., spacetime is AdS
- $\alpha \neq 0$ corresponds to relevant coupling for the CFT; deform. AdS

Holographic Renormalization Group

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The domain wall solution (d-dimensional Poincaré invariant)

$$ds^2 = e^{2\mathcal{A}(u)}\eta_{ij}dx^i dx^j + du^2, \quad \phi = \phi(u)$$

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- The β-function

$$\beta = \frac{d\lambda}{d\log E} = \frac{d\phi}{d\mathcal{A}}$$

Starting point - 5-dim background

Gravity + scalar field

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$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[R - \frac{1}{2}\, \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

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$$ds^2 = B^2(z)\left[-dt^2 + dx^2 + dy_1^2 + dy_2^2 + dz^2\right], \qquad \phi = \phi(z)$$

EOM:

Const.

$$\frac{\ddot{B}}{B} + \frac{2\dot{B}^2}{B^2} + \frac{3\dot{B}\dot{g}}{2B} + \frac{B^2V}{3} = 0,$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{3\dot{B}}{B}\right) - B^2\,\partial_\phi V = 0,$$

$$\frac{\ddot{B}}{B} - \frac{2\dot{B}^2}{B^2} + \frac{\dot{\phi}^2}{6} = 0$$

where $\dot{} = \partial/\partial z,\ \partial_\phi = \partial/\partial\phi$

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$$X = \frac{\dot\phi}{3}\,\frac{B}{\dot B}$$

$$\dot X = \frac{1}{3}\left(\ddot\phi\,\frac{B}{\dot B} - \dot\phi\,\frac{\ddot B B}{\dot B^2} + \dot\phi\right)$$

$$\boxed{\frac{3\dot X}{\dot\phi} = -4 + \frac{3XB^3\partial_\phi V}{g\dot\phi^2 B} + \frac{3}{2}X^2}$$

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$$\frac{3\dot X}{\dot\phi} = -\,4\,(1-\frac{3}{8}\,X^2)\left(1+\frac{1}{X}\,\frac{2\partial_\phi V}{2V}\right)$$

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Details see next page

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RGF equation:

$$\frac{dX}{d\phi} = -\,\frac{4}{3}\left(1-\frac{3}{8}\,X^2\right)\left(1+\frac{1}{X}\,\frac{\partial_\phi V}{V}\right)$$

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$$X = \frac{\dot\phi}{3}\,\frac{B}{\dot B}$$

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Compare with the method of super potential: $V = \frac{1}{8}W'(\phi)^2 - \frac{1}{3}W(\phi)^2$

Gravity + scalar field. Details, 1/2

$$\dot{X} = \frac{1}{3}\left\{\frac{B}{\dot{B}}\left(-\dot{\phi}\right)\left(\frac{\dot{g}}{g}+\frac{3\dot{B}}{B}\right)+\frac{B^3\partial_\phi V}{g\dot{B}}-\frac{\dot{\phi}B^2}{\dot{B}^2}\left(\frac{2\dot{B}^2}{B^2}-\frac{\dot{\phi}^2}{6}\right)+\dot{\phi}\right\}. \quad (*)$$

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Dividing on $\dot{\phi}$ gives

$$\frac{3\dot{X}}{\dot{\phi}} = -\,4-\frac{B\dot{g}}{\dot{B}g}+\frac{B^3\partial_\phi V}{g\dot{\phi}\dot{B}}+\frac{3}{2}\,X^2. \qquad (\#)$$

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Now our goal is to exclude $\dot{B}$ and $\dot{\phi}$ from the RHS of (#). We do this in 3 steps.

Gravity + scalar field. Details, 1/2

$$\dot{X} = \frac{1}{3}\left\{\frac{B}{\dot{B}}\left(-\dot{\phi}\right)\left(\frac{\dot{g}}{g} + \frac{3\dot{B}}{B}\right) + \frac{B^3\partial_\phi V}{g\dot{B}} - \frac{\dot{\phi}B^2}{\dot{B}^2}\left(\frac{2\dot{B}^2}{B^2} - \frac{\dot{\phi}^2}{6}\right) + \dot{\phi}\right\}. \quad (*)$$

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- We exclude $\dot{B}$ from (#) using relation $\dot{B} = \frac{\dot{\phi}B}{3X}$

$$\frac{3\dot{X}}{\dot{\phi}} = -\,4 + \frac{3XB^2\partial_\phi V}{g\dot{\phi}^2} + \frac{3}{2}\,X^2. \qquad (**)$$

Gravity + scalar field. Details, 2/2

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Step 2

Gravity + scalar field. Details, 2/2

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Step 2

- We get relation for $\dot{\phi}^2$ and substitute it into (**).

$$\frac{4\dot{\phi}^2}{9X^2} + \frac{B^2V}{3g} - \frac{\dot{\phi}^2}{6} = 0. \qquad (***)$$

We rewrite (***) as

$$\left(\frac{1}{6} - \frac{4}{9X^2}\right)\dot{\phi}^2 = \frac{2V}{6g}\,B^2 \qquad (****)$$

and from (****) get combination $g\dot{\phi}^2/B^2$

$$\frac{g\dot{\phi}^2}{B^2} = \frac{2V}{1 - \frac{8}{3X^2}}. \qquad (******)$$

Gravity + scalar field. Details, 2/2

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and from (****) get combination $g\dot{\phi}^2/B^2$

$$\frac{g\dot{\phi}^2}{B^2} = \frac{2V}{1 - \frac{8}{3X^2}}. \qquad (******)$$

Step 3

Substituting in the RHS of (**) the expression (******) we finally get

$$\frac{3\dot{X}}{\dot{\phi}} = -\,4\,(1 - \frac{3}{8}\,X^2)\Big(1 + \frac{1}{X}\,\frac{2\partial_\phi V}{2V}\Big).$$

Gravity + scalar field. Simple examples

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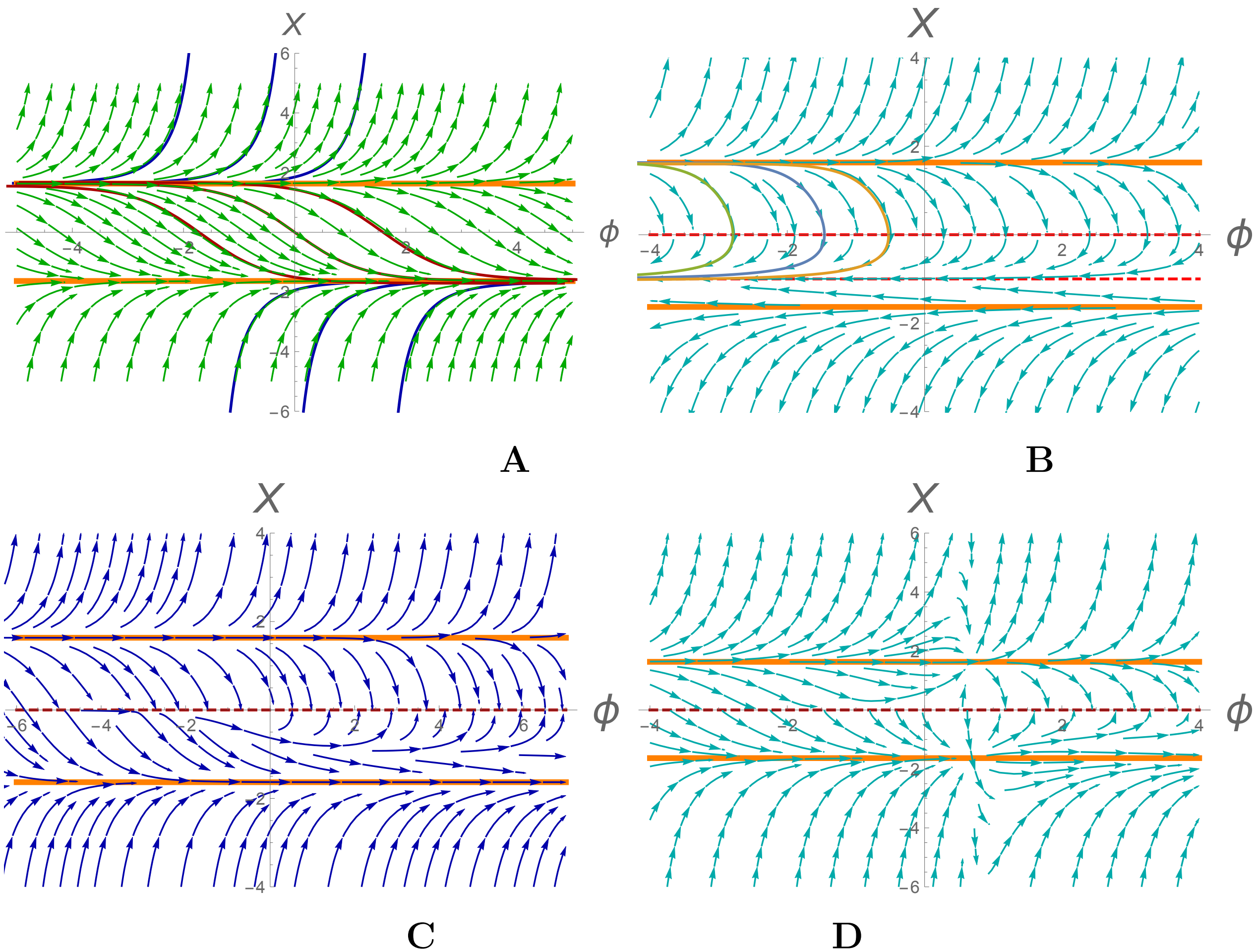
Dilaton gravity with one exponential potential

Chamblin Reall model $\quad V(\phi) = \exp(c\phi) + V_0$

$c = 1$

$$\phi = \phi_0 + \frac{1}{5}\Big[\left(2\sqrt{6}-3\right)\log\left|2\sqrt{6}-3X\right| + 6\log|X+1| - \left(3+2\sqrt{6}\right)\log\left|3X+2\sqrt{6}\right|\Big]$$

Stream-plots for vectors field



A) AdS, i.e. $V(\phi) = const$, B) $V(\phi) = \exp(c\phi)$, C) $V(\phi) = \exp(c\phi) + V_0$, $V_0 > 0$

D) $V(\phi) = \exp(c\phi) + V_0$, $V_0 < 0$.

Holographic RG Flow

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$$V_{AGP}(\varphi) = C_1 e^{2k\varphi} + C_2 e^{\frac{32}{9k}\varphi}$$

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IA, A.Golubtsova, G.Policastro, 1803.06764

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Generalization of Chamblin&Reall model,
hep/th. 9903225

Two exponent potential

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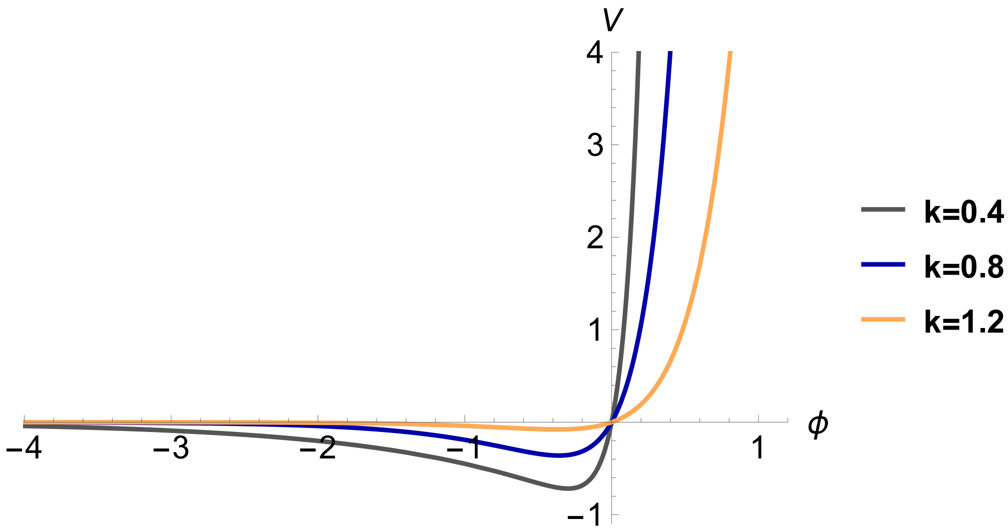

Exact solutions for the metric and the dilaton (zero tempertature)

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-dt^2 + d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2$$

$$\phi = -\frac{9k}{9k^2-16} \log F_1 + \frac{9k}{9k^2-16} \log F_2$$

Two exponent potential

Two exponent potential

- $$F_1 = \sqrt{\left|\frac{C_1}{2E}\right|}\sinh(\mu\,|u-u_{01}|), \mu = \sqrt{\left|\frac{3E}{2}(k^2-\frac{16}{9})\right|},$$
  $$F_2 = \sqrt{\left|\frac{C_2}{2E}\right|}\sinh(\frac{4}{3k}\mu\,|u-u_{02}|).$$

- $$F_1 = \sqrt{\frac{3}{4}(k^2-\frac{16}{9})C_1}(u-u_{01}), F_2 = \sqrt{\frac{4}{3}(\frac{16}{9k^2}-1)C_2}|u-u_{02}|$$
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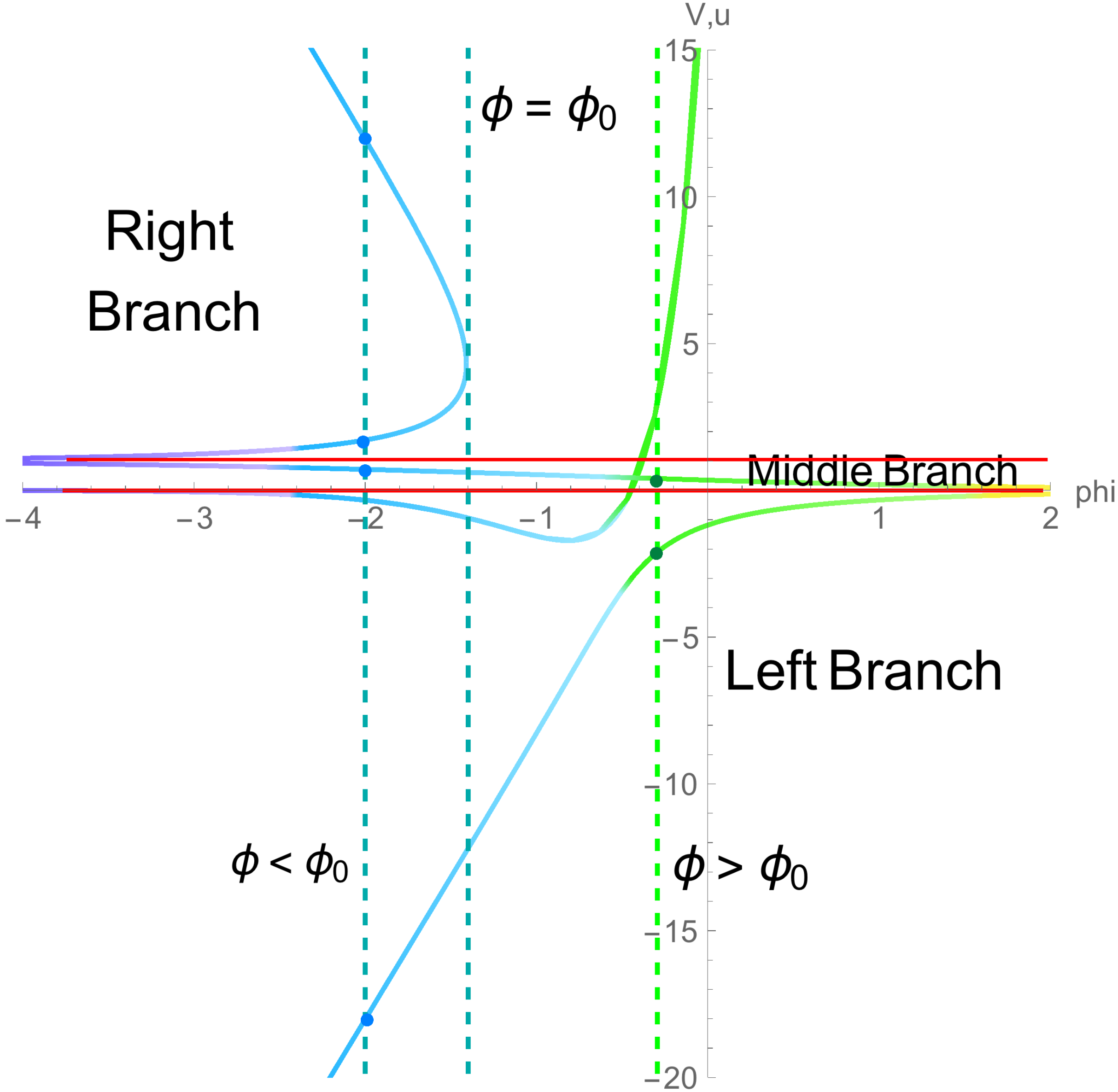
- types of RG flows: **1)** on $u \in (u_{02}, u_{01})$, **2)** $u \in (u_{01}, +\infty)$.
- boundaries of the backgrounds correspond to fixed points.
- $u_{01} = u_{02} \Rightarrow$ special RG flow : $u \in (u_0, +\infty)$ AdS UV fixed point

The solution for the dilaton - 3 regions!



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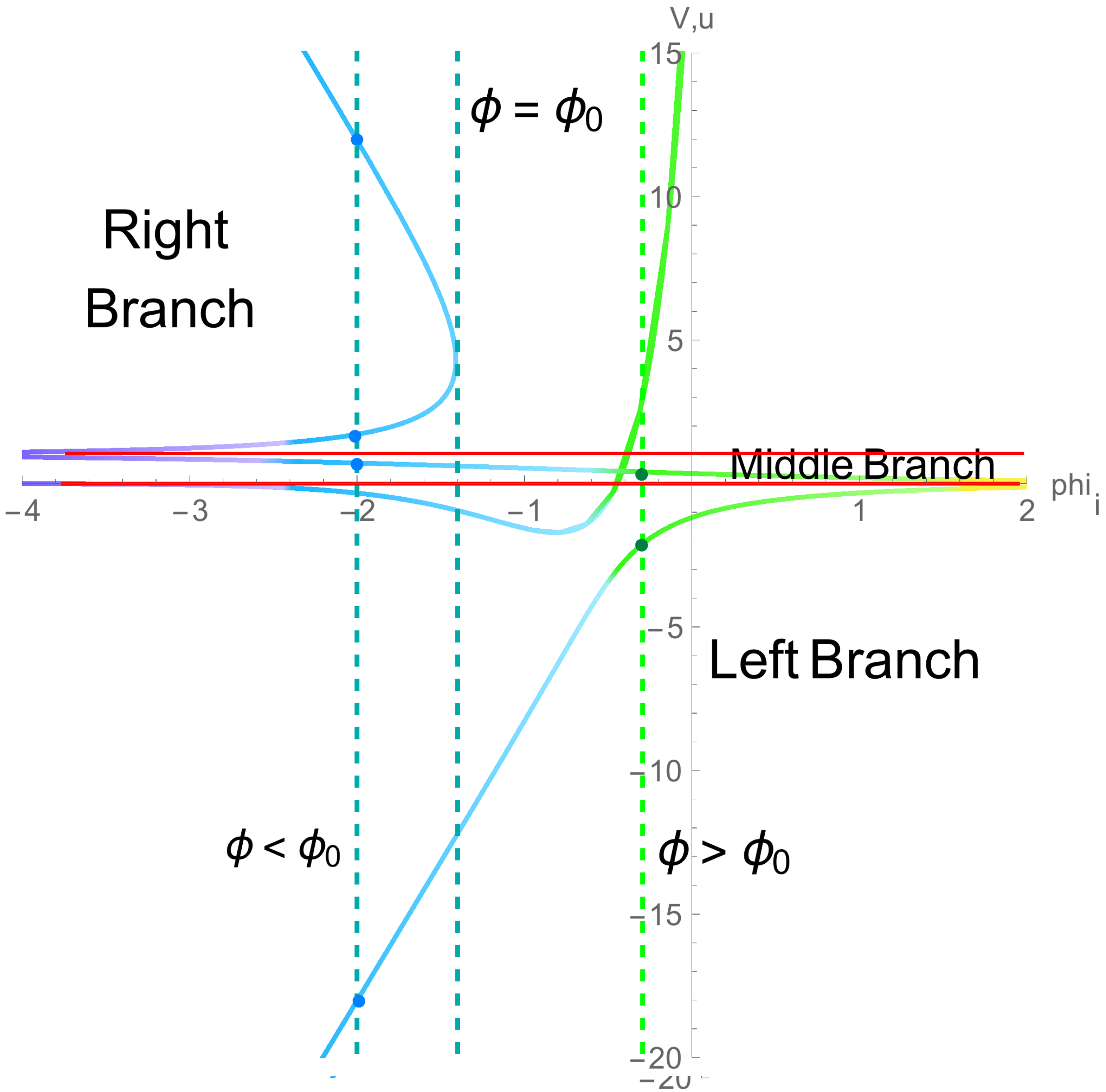

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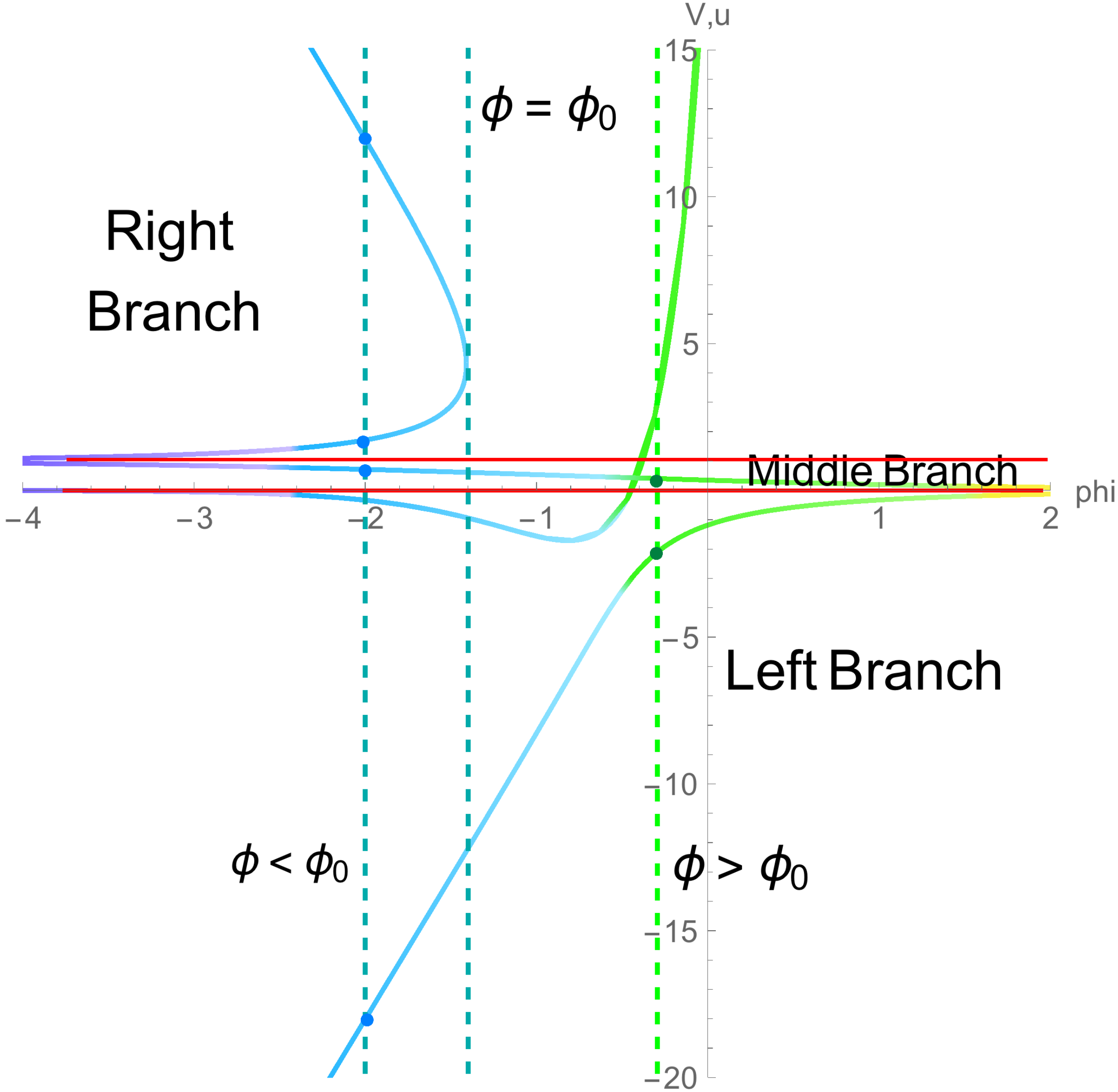

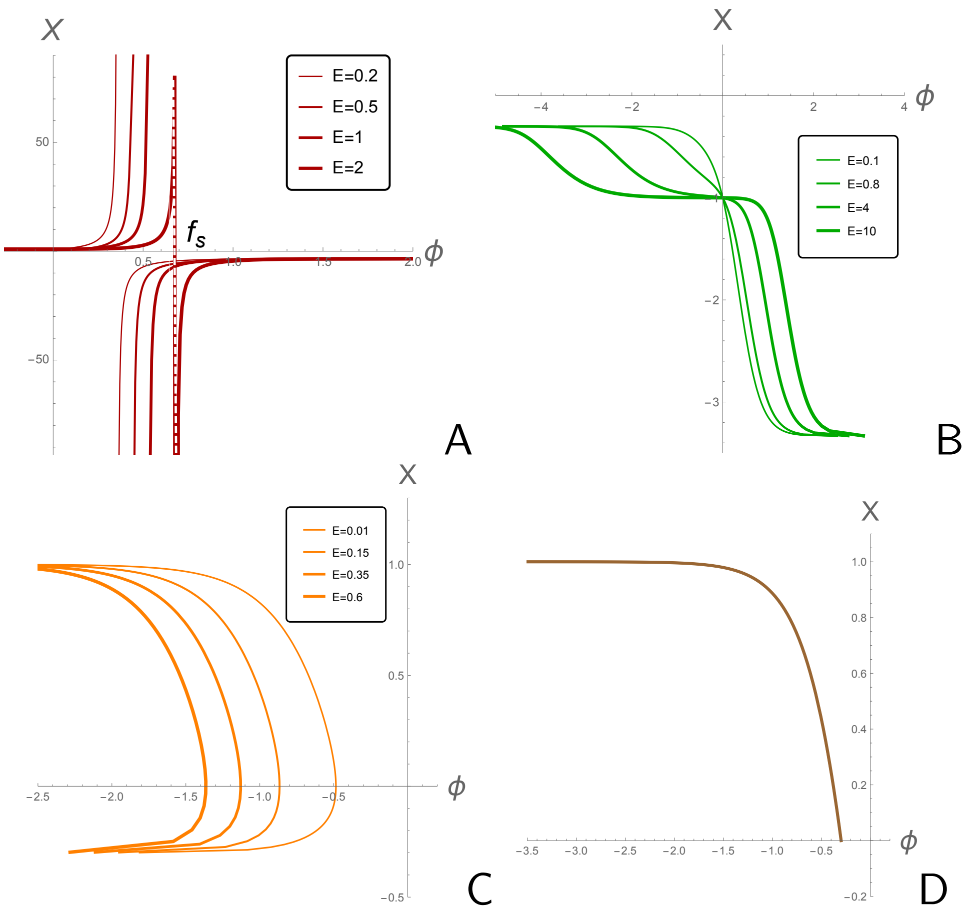

Figure: The behaviour of the X-function with the dependence on the dilaton plotted using the solutions for $\mathcal{A}$. A)left B) middle C)right D) $u_{01} = u_{02}$

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right)$$

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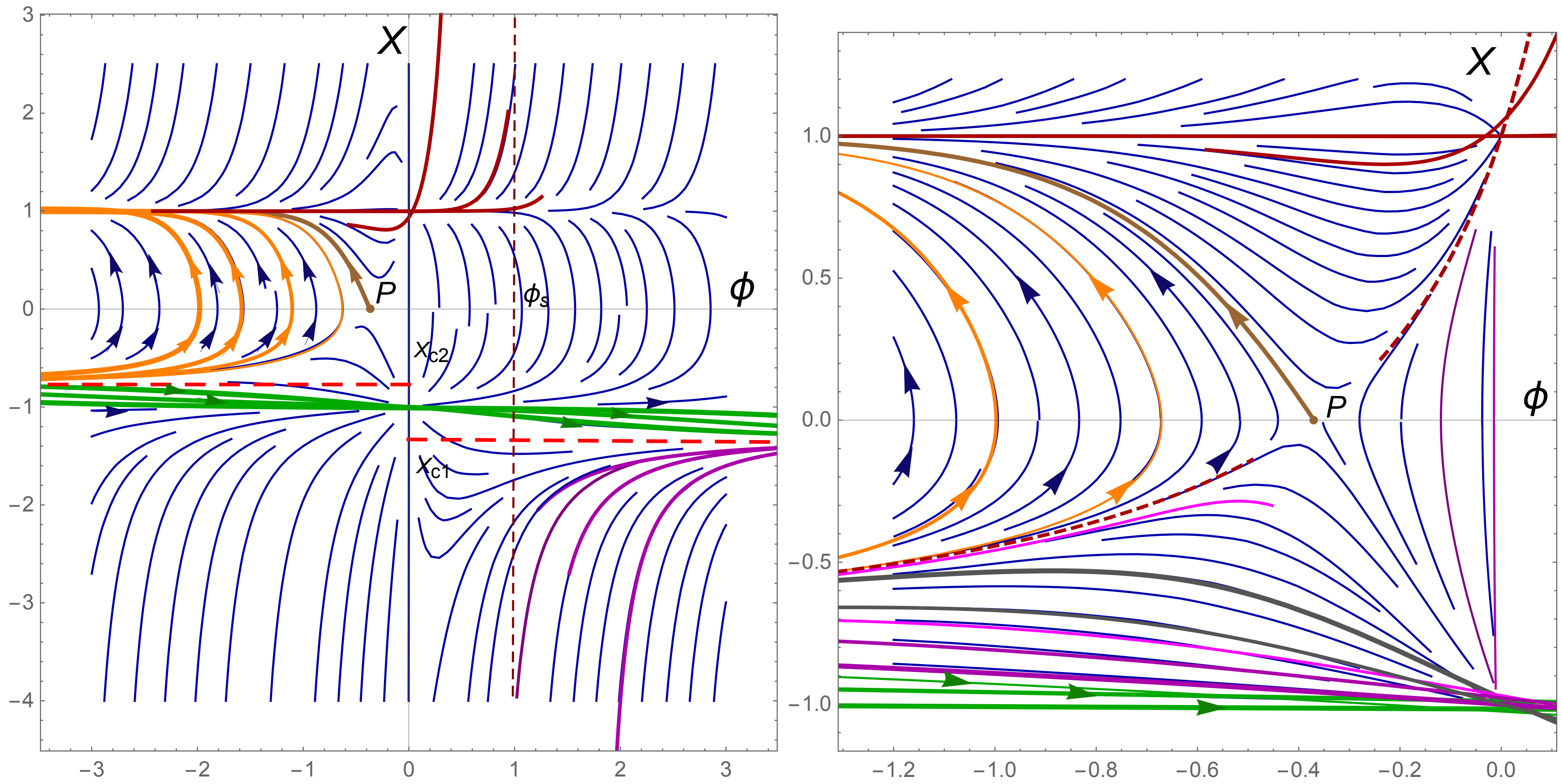
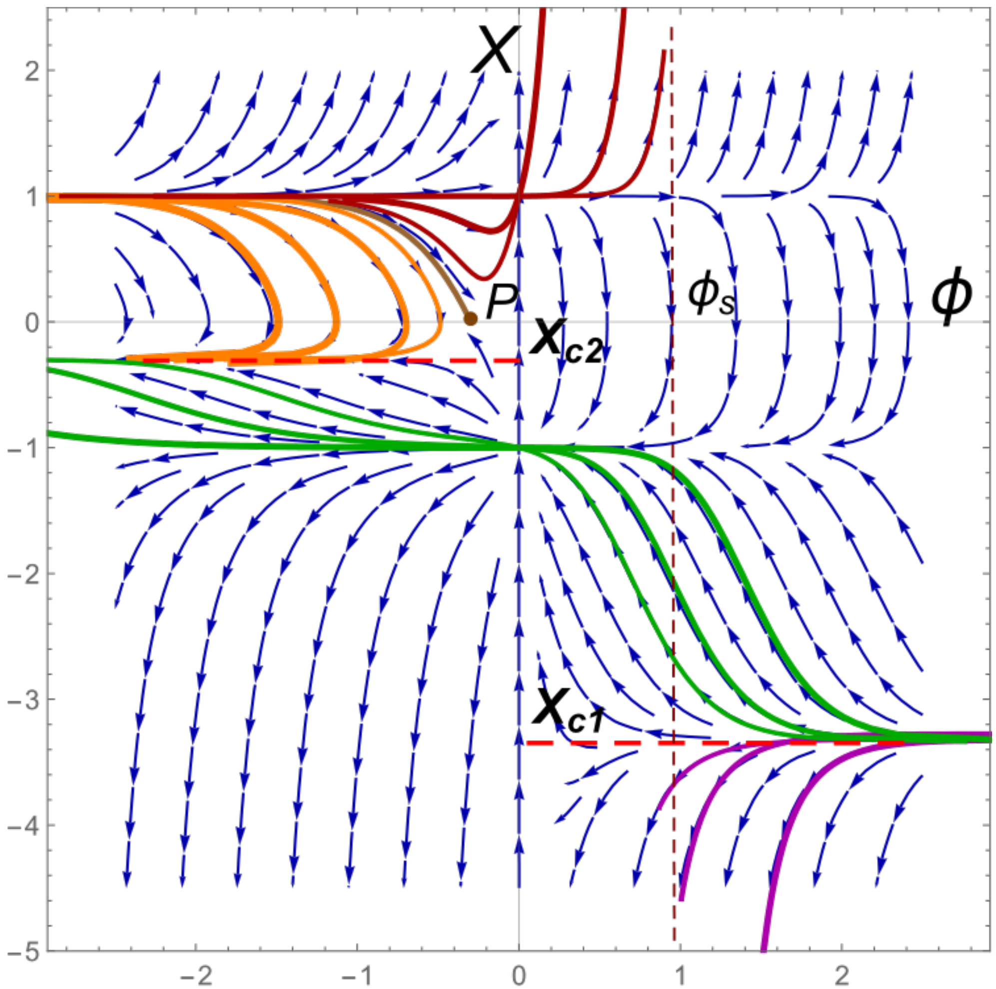
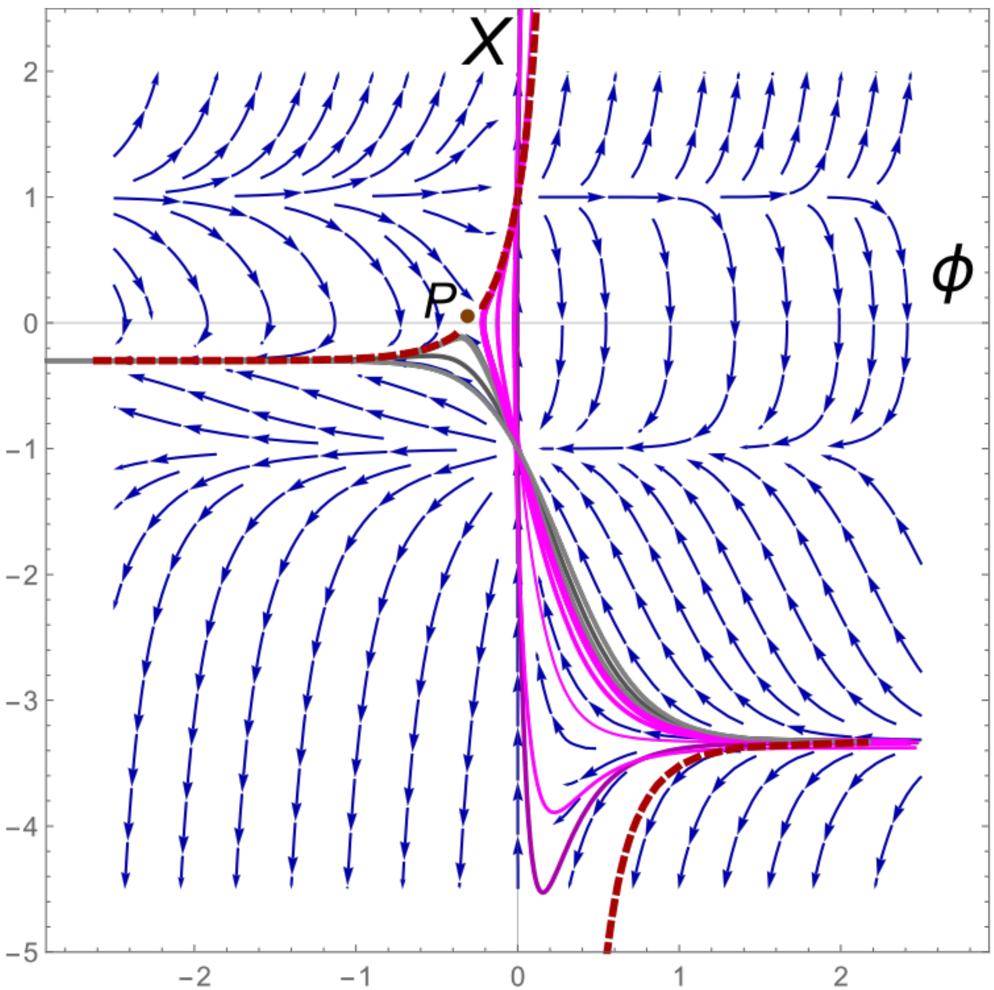


$\sinh -solutions$

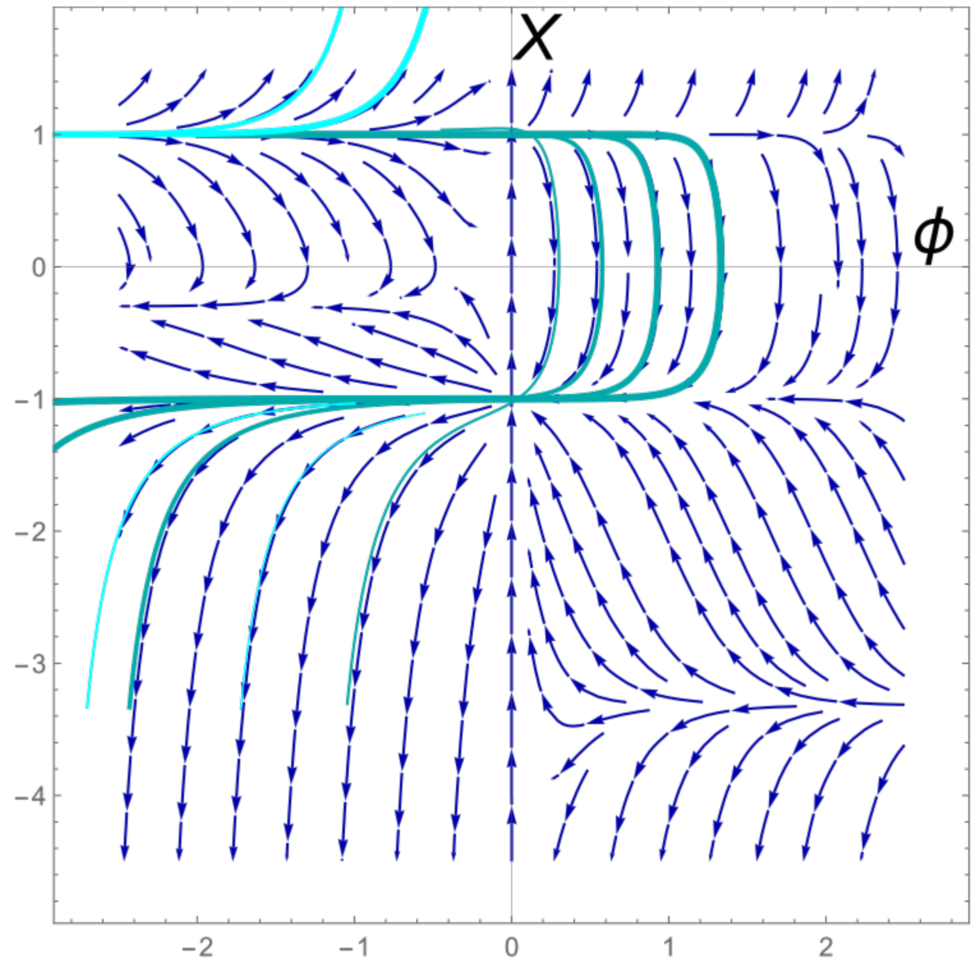


XPhivac.pdf

$$\sin - solutions$$



$$\cosh - solutions$$



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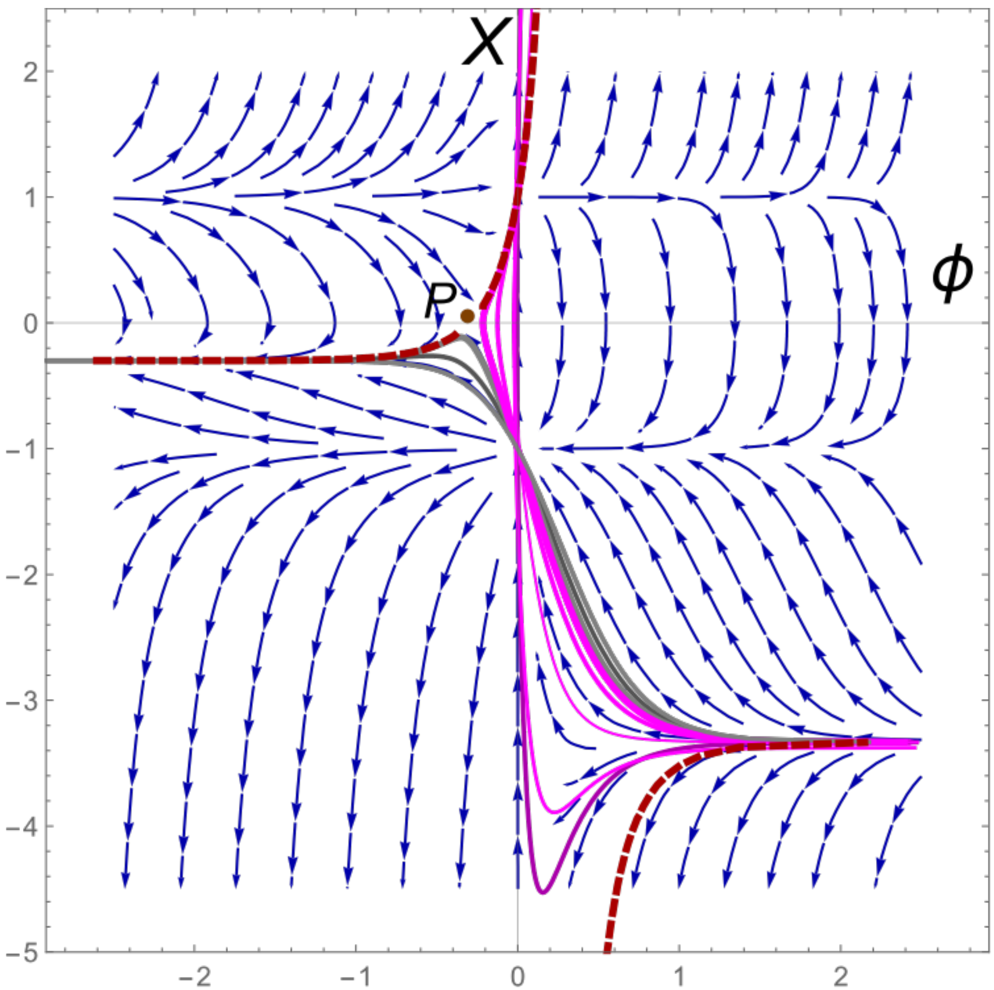

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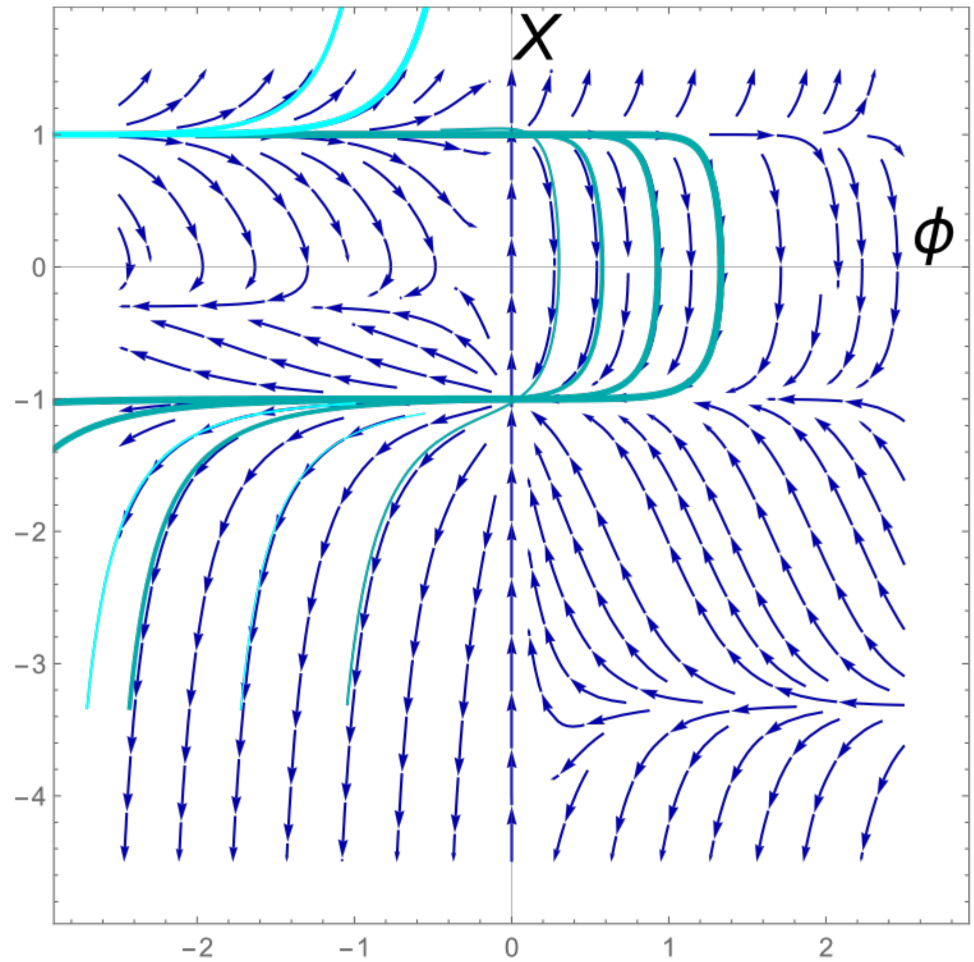

EOM as an autonomous system

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IA, A.Golubtsova, A. G Policastro,
M Skugoreva, Work in progress

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The domain wall ansatz

$$ds^2 = e^{2A(w)}(-dt^2 + d\vec{x}^2) + dw^2, \quad \phi = \phi(w)$$

Introduce new variables

$$X = \frac{1}{3}\frac{\dot{\phi}}{\dot{A}}, \qquad Z = \frac{1}{e^{\frac{2(16-9k^2)}{9k}\phi} + 1},$$

$$V = C_1\left(\frac{1-Z}{Z}\right)^{\frac{9k^2}{16-9k^2}} + C_2\left(\frac{1-Z}{Z}\right)^{\frac{16}{16-9k^2}}.$$

Then zeros of the potential are $Z = \frac{C_2}{C_2 - C_1}$.

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$$\frac{dZ}{dA} = \frac{2(9k^2-16)}{3k}(1-Z)ZX,$$

$$\frac{dX}{dA} = (X^2-1)\left(4X + \frac{16C_2 + Z(C_1 9k^2 - 16C_2)}{3k\Big(C_2 + Z(C_1 - C_2)\Big)}\right)$$

Autonomous system for holographic RG flows at T = 0

IA, AG, GP, MS'20

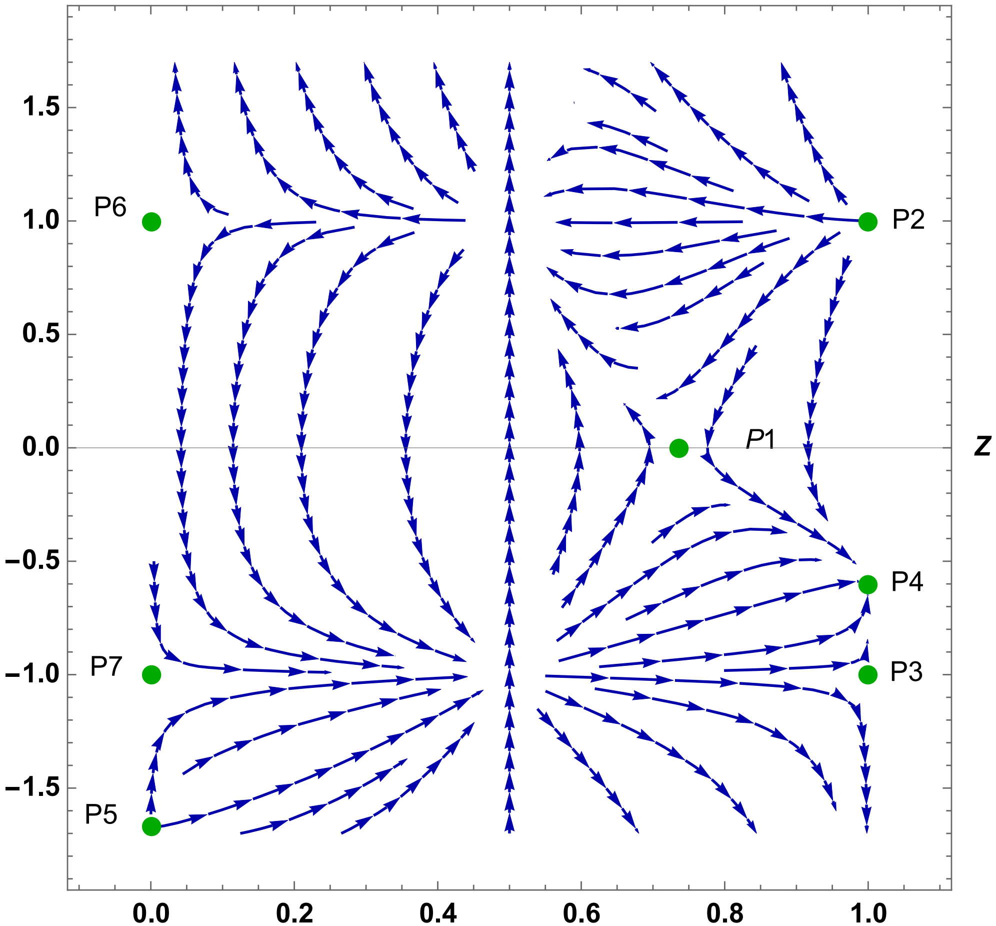

FIGURE: Phase portrait for the system at $T = 0$ with $k = 0.8$, the direction of arrows show with respect to increasing A

Autonomous system for holographic RG flows at $T = 0$

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To classify stability of equilibrium points we do the linear perturbation δZ and δX around each from the found critical points (Z_c, X_c) in order

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$$\frac{d}{dA}\begin{pmatrix} \delta Z \\ \delta X \end{pmatrix} = \mathcal{M}\begin{pmatrix} \delta Z \\ \delta X \end{pmatrix},$$

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$$\mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial Z} & \frac{\partial f}{\partial X} \\ \frac{\partial g}{\partial Z} & \frac{\partial g}{\partial X} \end{pmatrix}\Bigg|_{Z=Z_c, X=X_c}$$

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$$m_{11} = \frac{2(9k^2-16)}{3k}(1-2Z_c)X_c, \quad m_{12} = \frac{2(9k^2-16)}{3k}(1-Z_c)Z_c$$
$$m_{21} = \frac{C_1C_2({X_c}^2-1)(9k^2-16)}{3k(C_2+Z_c(C_1-C_2))^2},$$
$$m_{22} = 4(3{X_c}^2-1) + 2X_c\left[\frac{16C_2+Z_c(C_1 9k^2-16C_2)}{3k\Big(C_2+Z_c(C_1-C_2)\Big)}\right].$$

Backup

Backup

1. $X = 0$, $Z = \frac{16C_2}{16C_2 - 9k^2 C_1}$. $V = const$. The eigenvalues $\lambda_1 = 4$, $\lambda_2 = -8$, a saddle. The scalar field and the metric

$$\phi_{P_1} = \frac{9k}{2(16 - 9k^2)} \ln\left(-\frac{9k^2}{16}\frac{C_1}{C_2}\right), \quad \frac{C_1}{C_2} < 0,$$
$$ds^2 = e^{2\mathcal{C}(w - w_0)}\left(-dt^2 + d\vec{x}^2\right) + dw^2,$$

where w_0 is a constant of integration, $\mathcal{C} = \pm\sqrt{\frac{-C_1(16-9k^2)}{192}}\left(-\frac{9k^2}{16}\frac{C_1}{C_2}\right)^{\frac{9k^2}{2(16-9k^2)}}$

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2. $X = 1$, $Z = 1$. $V \to 0$. The eigenvalues $\lambda_{1,2} = \frac{2}{3k}(4 + 3k)(2 \pm |2 - 3k|)$. An unstable node. The scalar field and the metric

$$\phi(w) = \frac{3}{4} \ln\left|\frac{w - w_0}{w_2}\right| \to -\infty \quad \text{for} \quad w \to w_0,$$
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3. $X = -1$, $Z = 1$. $V \to 0$, $\lambda_1 = 2(4 - 3k) > 0$, $\lambda_2 = \frac{2}{3k}(9k^2 - 16) < 0$ – a saddle.

$$\phi(w) = -\frac{3}{4}\ln\left|\frac{w}{w_2}\right| \to -\infty \text{ for } w \to +\infty, \quad ds^2 = \left|\frac{w}{w_1}\right|^{1/2}\left(-dt^2 + d\vec{x}^2\right) + dw^2,$$

where w_1, w_2 are constants.

Backup

4. $X = -\frac{3k}{4}$, $Z = 1$,$V \to 0$. A stable node: $\lambda_1 = \frac{1}{4}(9k^2 - 16), \lambda_2 = \frac{1}{2}(9k^2 - 16)$

$$\phi(w) = -\frac{1}{k}\ln\left|\frac{w}{w_2}\right| \to -\infty \text{ for } w \to +\infty, \quad ds^2 = \left|\frac{w}{w_1}\right|^{\frac{8}{9k^2}}(-dt^2 + d\vec{x}^2) + dw^2,$$

5. $X = -\frac{4}{3k}$, $Z = 0$. $\lambda_1 = \frac{8}{9k^2}(16 - 9k^2), \lambda_2 = \frac{4}{9k^2}(16 - 9k^2)$. An unstable node. $V \to +\infty$.

$$\phi = -\frac{9k}{16}\ln\left|\frac{w - w_0}{w_2}\right| \to +\infty \text{ for } w \to w_0, \quad ds^2 = \left|\frac{w - w_0}{w_1}\right|^{\frac{9k^2}{32}}(-dt^2 + d\vec{x}^2) + dw^2$$

where w_0, w_1 and w_2 are some constants of integration.

6. $X = 1$, $Z = 0$, $V \to +\infty$. $\lambda_1 = \frac{8}{3k}(3k + 4) > 0$, $\quad \lambda_2 = \frac{2}{3k}(9k^2 - 16) < 0$, a saddle. The asymptotic form of the metric and the scalar field

$$\phi = \frac{3}{4}\ln\left|\frac{w}{w_2}\right| \to +\infty \text{ for } w \to +\infty, \quad ds^2 = \left|\frac{w}{w_1}\right|^{\frac{1}{2}}(-dt^2 + d\vec{x}^2) + dw^2.$$

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7. $X = -1$, $Z = 0$. $\lambda_1 = \frac{2}{3k}(16 - 9k^2) > 0, \lambda_2 = \frac{8}{3k}(3k - 4) < 0$, a saddle. $V(\phi) \to +\infty$.

$$\phi = -\frac{3}{4}\ln\left|\frac{w - w_0}{w_2}\right| \to +\infty \text{ for } w \to w_0, \quad ds^2 = \left|\frac{w - w_0}{w_1}\right|^{\frac{1}{2}}(-dt^2 + d\vec{x}^2) + dw^2.$$

Backup

Types of fixed points

P_1	P_2	P_3	P_4	P_5	P_6	$P7$
$V = const$	$V \to 0$	$V \to 0$	$V \to 0$	$V \to \infty$	$\times$	$\times$
ustable	stable	unstable	ustable	stable	$\times$	$\times$
UV	IR	UV	UV	IR	$\times$	$\times$

(We take into account that A should have the opposite direction)
Examples of flows:

- $P_1(AdS$ -UV)-P_2(hyperscaling vioation in IR)
- P_4 (hypersc. v. in UV)- P_2 (hypersc. v. in IR), bouncing solution AGP'18
- P_4 (hypersc. v. in UV)- P_5 (hypersc. v. in IR) confining solution (AG,Ngyuen Vu'1906.12316)

Black holes in terms of w-coordinates

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$$ds^2 = e^{2A(w)}(-f(w)dt^2 + d\vec{x}^2) + \frac{dw^2}{f(w)},$$

and the dilaton is

$$\lambda = e^{\phi(w)}.$$

The function f is a so-called blackening function. It is convenient to introduce a new variable

$$g = \ln f.$$

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The equations of motion with respect to w-variable are

$$\begin{aligned} 12\dot{A}^2 + 3\dot{A}\dot{g} - \frac{4}{3}\dot{\phi}^2 + e^{-g}V &= 0, \\ \ddot{A} + \frac{4}{9}\dot{\phi}^2 &= 0, \\ \dot{g} + \frac{\ddot{g}}{\dot{g}} + 4\dot{A} &= 0, \\ \ddot{\phi} + 4\dot{A}\dot{\phi} + \dot{g}\dot{\phi} - \frac{3}{8}e^{-g}\frac{dV}{d\phi} &= 0. \end{aligned}$$

Holographic Isotropic zero μ RG Flow

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Zero chemical potential $H_1 = 0$ Non-zero temperature $Y \neq 0$

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$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2 + 1\right)\left(1 + \frac{3}{8}\frac{V'}{VX}\right)$$

$$\frac{dY}{d\phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\frac{Y}{X}$$

Holographic Isotropic zero μ RG Flow

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Gursoy, Kiritsis, Mazzanti, Nitti, arXiv:0812.0792

Autonomous equations at finite T

Autonomous equations at finite T

New variables

$$X = \frac{1}{3}\frac{\dot{\phi}}{\dot{A}(w)}, \quad Y = \frac{1}{4}\frac{\dot{g}}{\dot{A}}$$

and also

$$Z = \frac{1}{e^{\frac{2(16-9k^2)}{9k}\phi} + 1}, \quad V = C_1\left(\frac{1-Z}{Z}\right)^{\frac{9k^2}{16-9k^2}} + C_2\left(\frac{1-Z}{Z}\right)^{\frac{16}{16-9k^2}}$$

Autonomous equations at finite T

New variables

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Taking derivatives with respect to A from X, Y, Z we obtain

$$\frac{dZ}{dA} = \frac{2(9k^2-16)}{3k}(1-Z)ZX,$$

$$\frac{dX}{dA} = (X^2-1-Y)\frac{16C_2 + Z(9k^2C_1 - 16C_2) + 12kX(Z(C_1-C_2)+C_2)}{3k((C_1-C_2)Z+C_2)}$$

$$\frac{dY}{dA} = 4Y(X^2-1-Y).$$

Fixed points

Fixed points

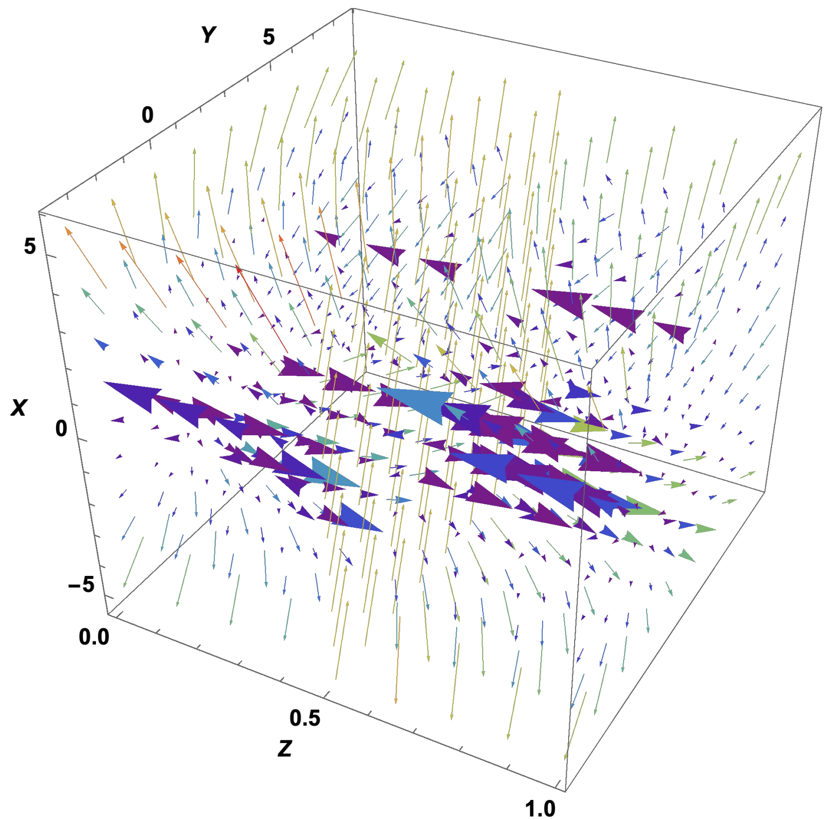

FIGURE: 3D phase portrait for the system at T with $k = 0.8$, the direction of arrows show the increasing energy scale (A)

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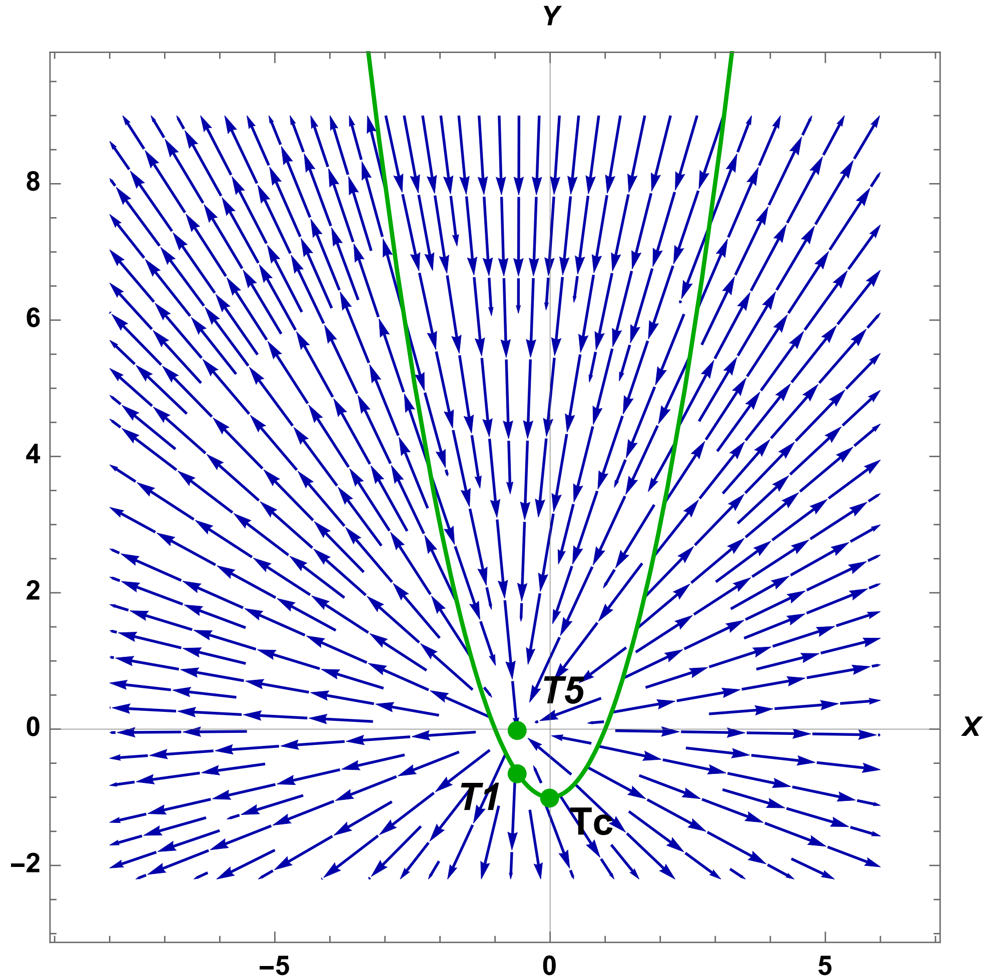

FIGURE: Phase portrait for the thermal system with $k = 0.8$, projection to $X - Y$-plane with $Z = 1$

Backup

Fixed points at finite T

1. $X \in (-\infty; +\infty)$, $Y = X^2 - 1$, $Z = 1$. $\phi \to -\infty$, $V \to 0$. The eigenvalues

$$\lambda_1 = \frac{2}{3k}(16 - 9k^2)X, \quad \lambda_2 = 0, \quad \lambda_3 = 4(X^2 + \frac{3k}{2}X + 1).$$

Since $0 < k < 4/3$ $\lambda_1 < 0$ $X < 0$ and $\lambda_1 > 0$ for $X > 0$, λ_3 is always positive for $X \in (-\infty, \infty)$. For negative X we have a saddle while for positive this fixed point is unstable node (a saddle-node bifurcation). For $X < 0$ the β-function is negative while for $X > 0$ it is positive.

$$\frac{dX}{dA} = (X^2 - 1 - Y)(3k + 4X),$$

2. $X \in (-\infty; +\infty)$, $Y = X^2 - 1$, $Z = 0$.

$$\lambda_1 = \frac{2}{3k}(9k^2 - 16)X, \lambda_2 = 0, \lambda_3 = 4\left(X + \frac{\sqrt{16 - 9k^2} + 4}{3k}\right)\left(X - \frac{\sqrt{16 - 9k^2} - 4}{3k}\right).$$

$\lambda_1 > 0$ with $X < 0$, $\lambda_1 < 0$ with $X > 0$, $\lambda_3 < 0$ for $X \in \left(-\frac{\sqrt{16-9k^2}+4}{3k}; \frac{\sqrt{16-9k^2}-4}{3k}\right)$, while for $X \in \left(-\infty, -\frac{\sqrt{16-9k^2}+4}{3k}\right) \cup \left(\frac{\sqrt{16-9k^2}-4}{3k}, +\infty\right)$ $\lambda_3 > 0$. Choosing k and X

we can get **a)** both unstable and stable nodes, **b)** saddle fixed points.

HRGF for non-zero chemical potential and anisotropic metric

5-dim Background

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Einstein-dilaton-two-Maxwell

5-dim Background

Einstein-dilaton-two-Maxwell

$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

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Coupling constant $\lambda = e^{\phi}$

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Holographic Anizotropic RenormGroup Flow

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Thank you for your attention