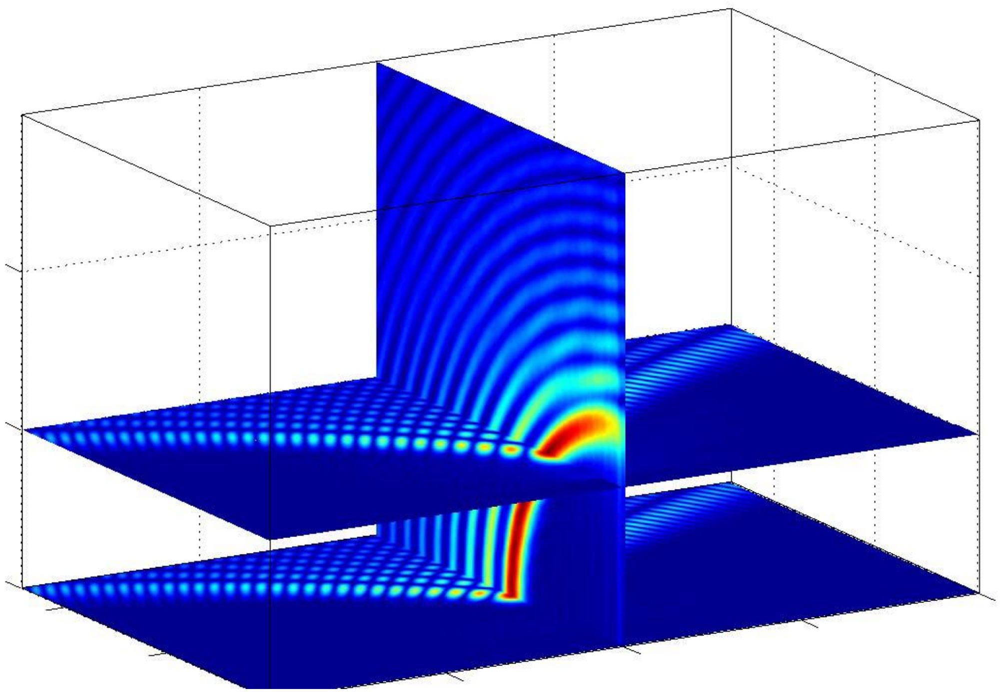
Weird light propagation

Yiqi Zhang and Milivoj R. Belić

Xi'an Jiaotong University and TAMUQ, Doha



Belgrade, Serbia, October 2021

Outline

- Accelerating diffractionless light beams?
- Airy wave-packets in quantum mechanics and optics
- Other accelerating beams: Mathieu, Weber, Fresnel...
- Accelerating beams in photonic crystals
- Beams in fractional Schrödinger equation

Light travels in straight lines - Right?

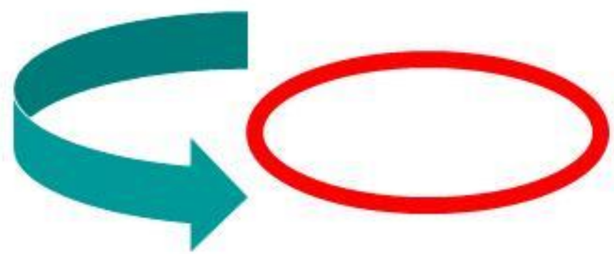
Courtesy of D. Christodoulides

Euclid of Alexandria
325-265 BC

Euclid's Optics
~ 300 BC

Yes, but…

Yet, can light bend ??

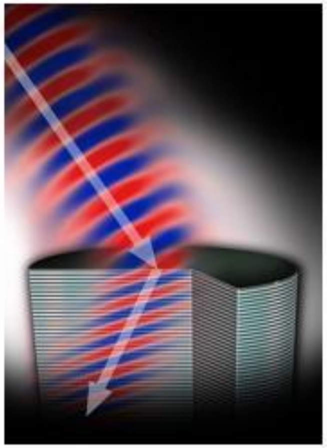


In arts – yes!
In physics?

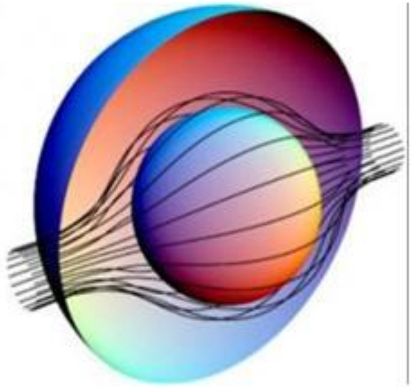
Can light bend ??

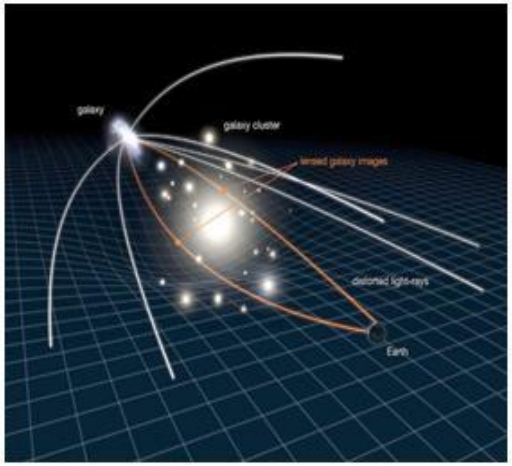
Refraction

Negative refraction



Cloaking



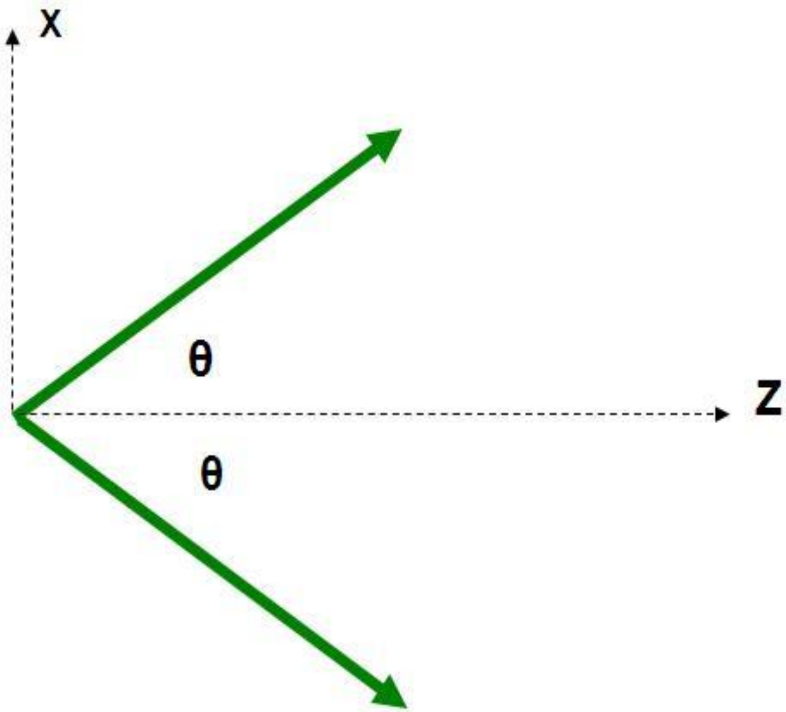


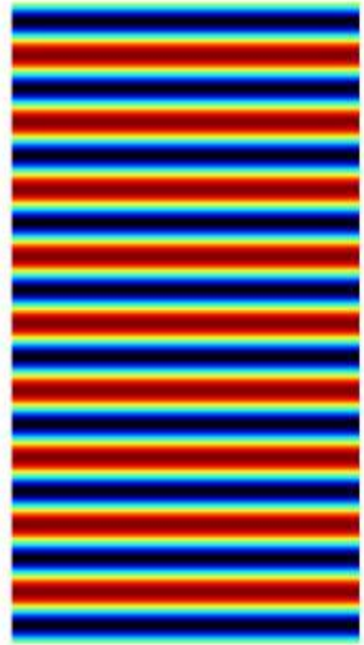
Gravitational lensing

But, can light accelerate?

Sure!

How about Diffraction-free patterns?





<u>Simple pattern</u>: Two-wave mixing

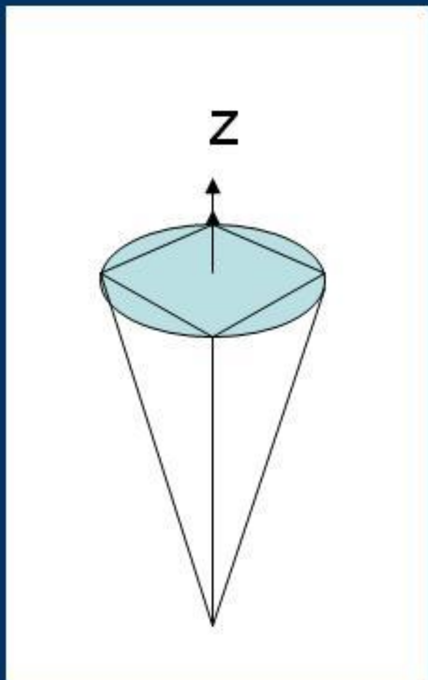
$$E = E_0 \exp[i(k_x x + k_z z)] + E_0 \exp[i(-k_x x + k_z z)]$$

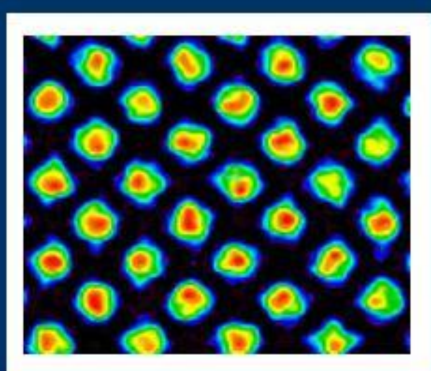
$$E = 2E_0 \cos(k_x x)\exp[ik_z z]$$

$$I = |E|^2 = 4E_0^2 \cos^2(k_x x)$$

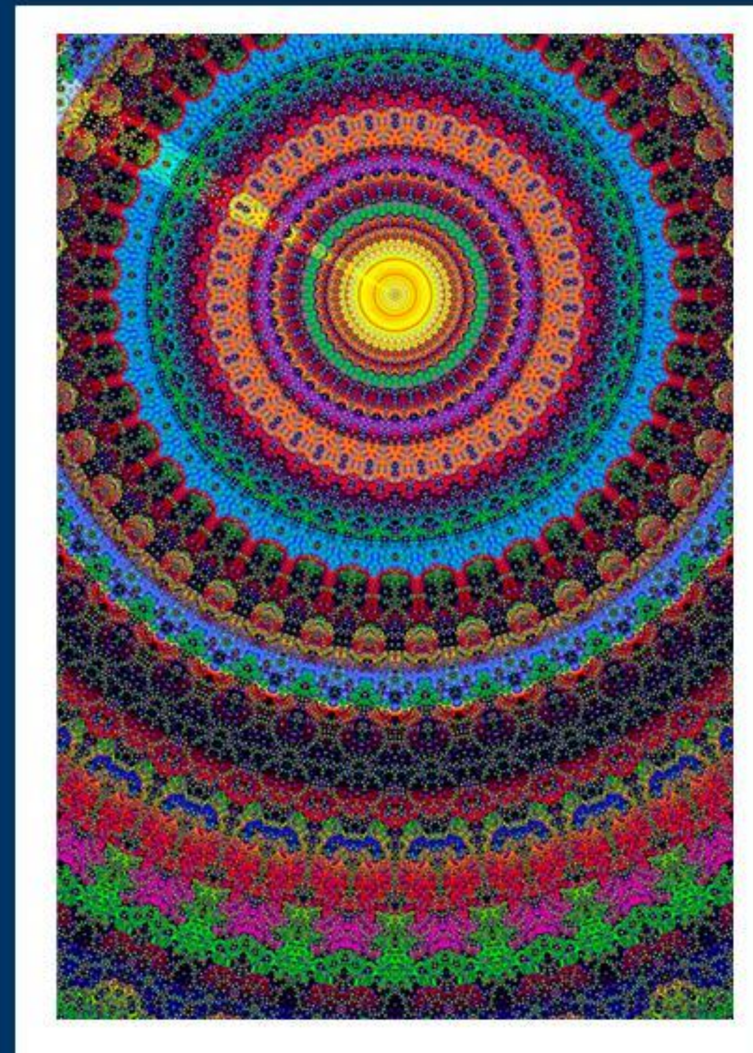
Propagating diffraction-free waves often accelerate!

Non-diffracting beams
- conical plane wave superposition





4-waves



21 waves

It started all in quantum mechanics: Airy wave

M.V. Berry and N. L. Balazs, "Nonspreading wave packets,"
***Am. J. Phys*. 47, 264 (1979)**

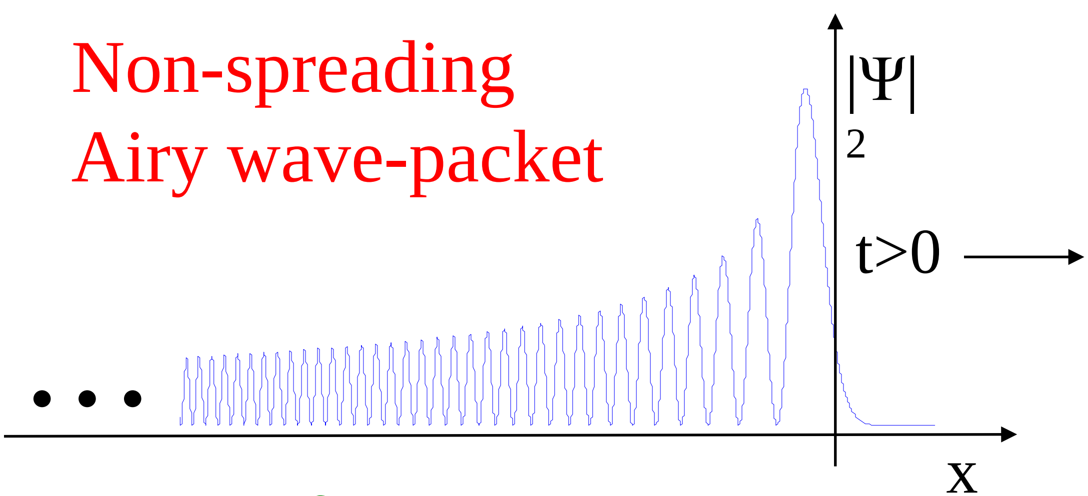
Free-particle Schrödinger equation

$$\boxed{i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=0}$$

Unique Airy wave-packet solution

$$\psi(x,t)=\mathrm{Ai}\left[\frac{B}{\hbar^{2/3}}\left(x-\frac{B^3t^2}{4m^2}\right)\right]e^{(iB^3t/2m\hbar)[x-(B^3t^2/6m^2)]}.$$

Non-spreading
Airy wave-packet



acceleration

Where from?

Courtesy of Ady Arie

Sir George Biddel Airy, 1801-1892

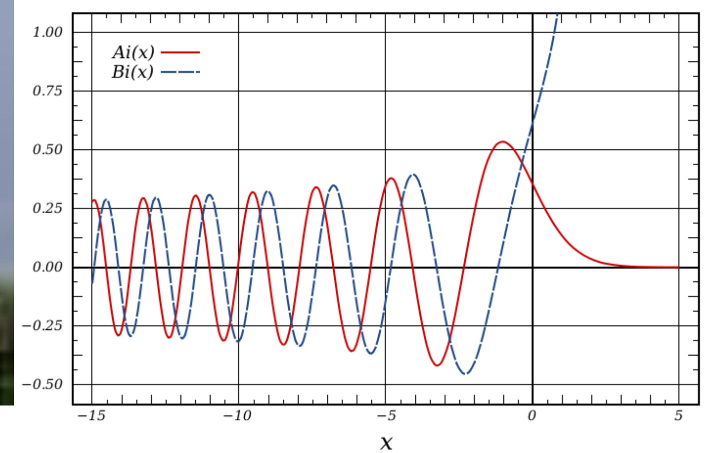
The Airy function is named after the British astronomer Airy, who introduced it during his studies of rainbows.

Equation:

y''(x)=xy

Solution:

$$y(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ipx}e^{ip^3/3}\,dp \equiv \mathrm{Ai}(x)$$



How come?

Solution? Go to inverse space, young man

- Airy equation

$$\frac{d^2y}{dx^2} = xy$$

- Fourier transform

$$\hat{y}(p) = \int_{-\infty}^{\infty} e^{-ipx} y(x)\, dx$$

- Derivatives

$$\frac{d}{dx} \mapsto +ip, \qquad \frac{d}{dp} \mapsto -ix$$

- Equation in the inverse space

$$-p^2\hat{y} = i\frac{d\hat{y}}{dp}$$

- Solution

$$\hat{y}(p) = e^{ip^3/3} \qquad y(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ipx} e^{ip^3/3}\, dp \equiv \mathrm{Ai}(x)$$

- Paraxial wave equation

$$i\partial_z u(\rho, z) = -\Delta u(\rho, z)$$

GO TO INVERSE SPACE

$$\frac{\partial u}{\partial x} \to ik_x\tilde{u} \quad \mathrm{grad}\, u \to i\vec{q}\tilde{u} \quad \Delta u \to -q^2\tilde{u}$$

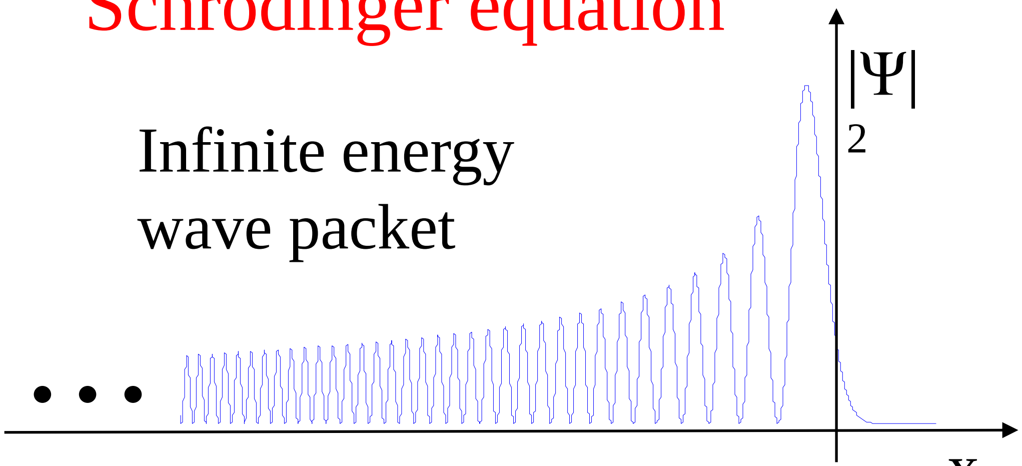
$$i\frac{\partial\tilde{u}}{\partial z} = q^2\tilde{u} \Rightarrow \tilde{u} = e^{-iq^2z}\tilde{u}_0 \qquad u(z,\rho) = (FT)^{-1}\exp(-iq^2z)(FT)u(0,\rho)$$

From Quantum Mechanics to Optics

$$\boxed{i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=0}$$

Free particle
Schrödinger equation

Infinite energy
wave packet



Berry and Balzas, 1979

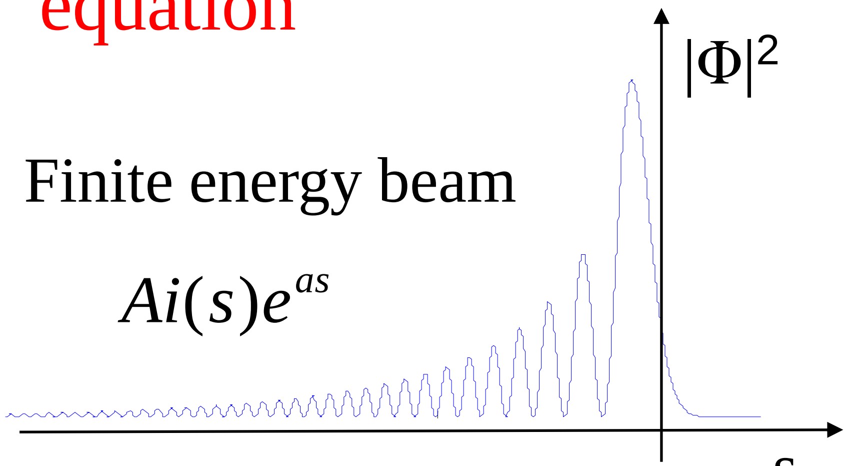
- Non diffracting
- Freely accelerating

$$\boxed{i\frac{\partial\Phi}{\partial\xi}+\frac{1}{2}\frac{\partial^2\Phi}{\partial s^2}=0}$$

Scaled paraxial wave
equation

Finite energy beam

$Ai(s)e^{as}$



Siviloglou and Christodulides, 2007

- Nearly non diffracting
- Freely accelerating

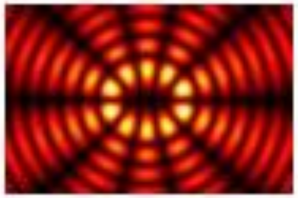
Siviloglou, Broky, Dogariu, & Christodoulides, *Phys. Rev. Lett.* **99**, 213901 (2007).

Nondiffracting optical waves

2D

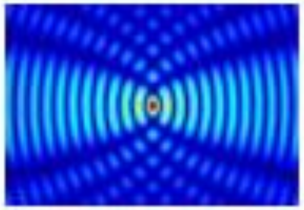
Bessel beam

(Cylindrical coordinate)



Mathieu beam

(Elliptic coordinate)



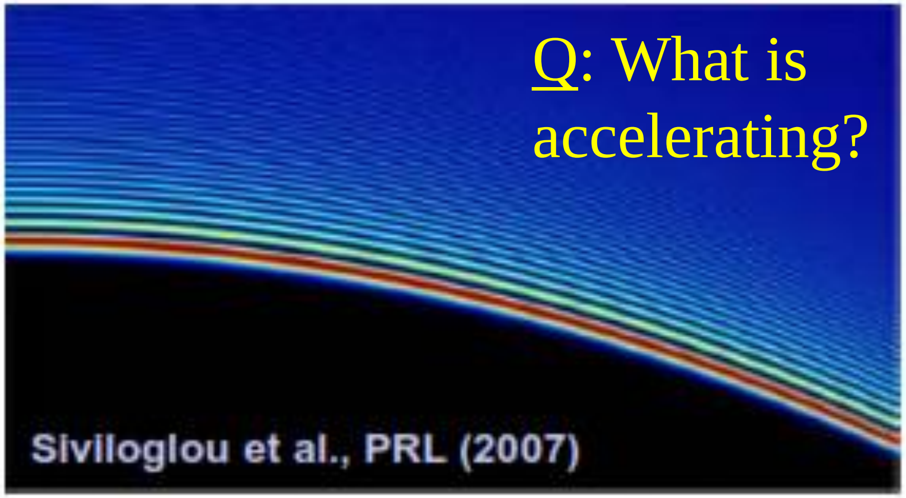
Parabolic beam

(Parabolic coordinate)

Airy beam in 1D

$$\psi(s,\xi) = Ai\left(s - \frac{\xi^2}{4}\right)\exp\left[i\left(\frac{s\xi}{2} - \frac{\xi^3}{12}\right)\right]$$

Free fall



- The only possible nondiffracting wave in 1D
- Self-healing property
- Transverse momentum (self-bending)

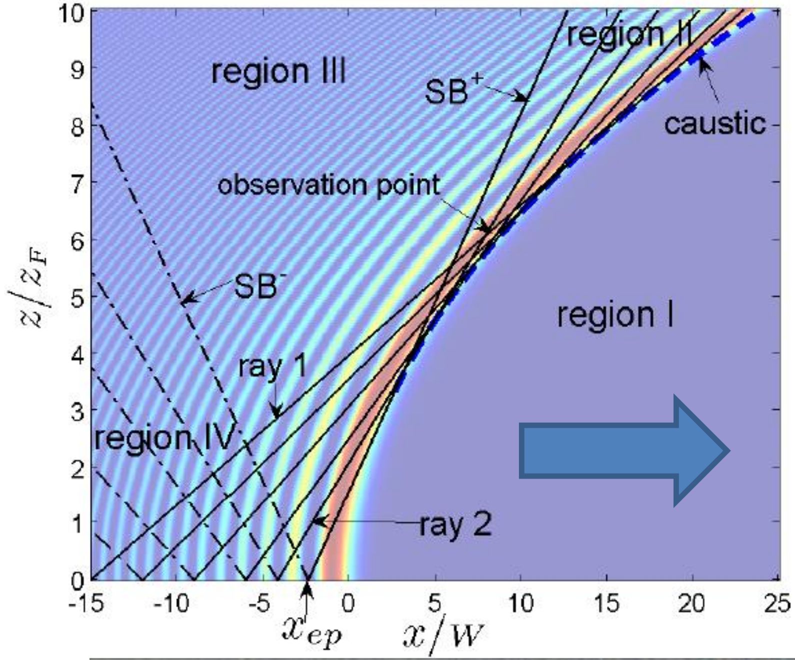
Courtesy of Z. Chen

In optics, Airy beam is a manifestation of caustic

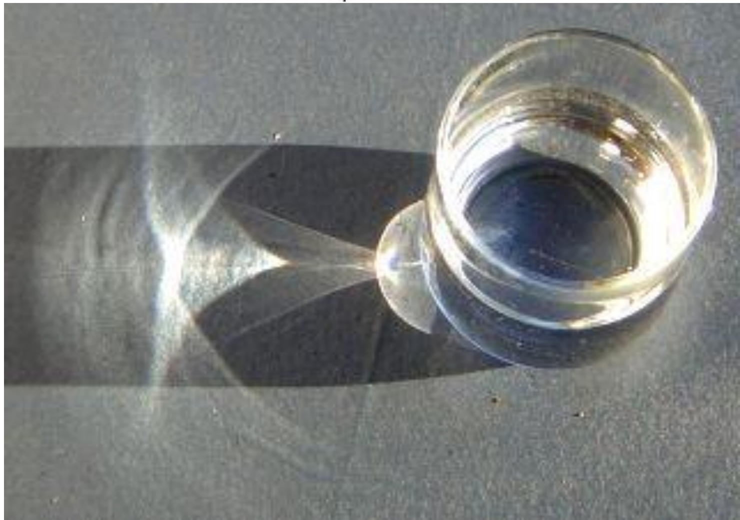
Caustic: An envelope of light rays reflected or refracted by a curved surface or object or the projection of that envelope of rays on another surface.

In a ray description, the rays are tangent to the parabolic line but do not cross it.

The Airy beam is a beam with curving propagation trajectories of its lobes.

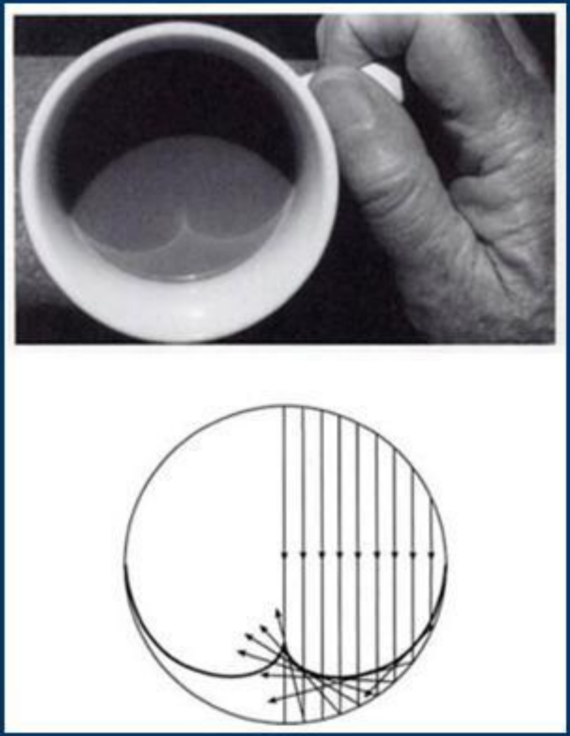


Curved caustics in every day life



Kaganovsky and Heyman, *Opt. Exp.* **18**, 8440 (2010)

Caustics are everywhere

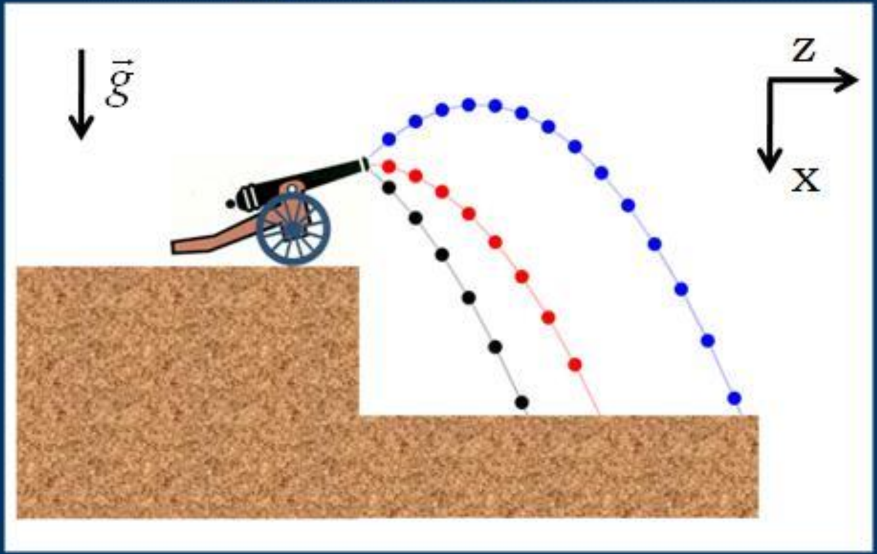


Pool caustics

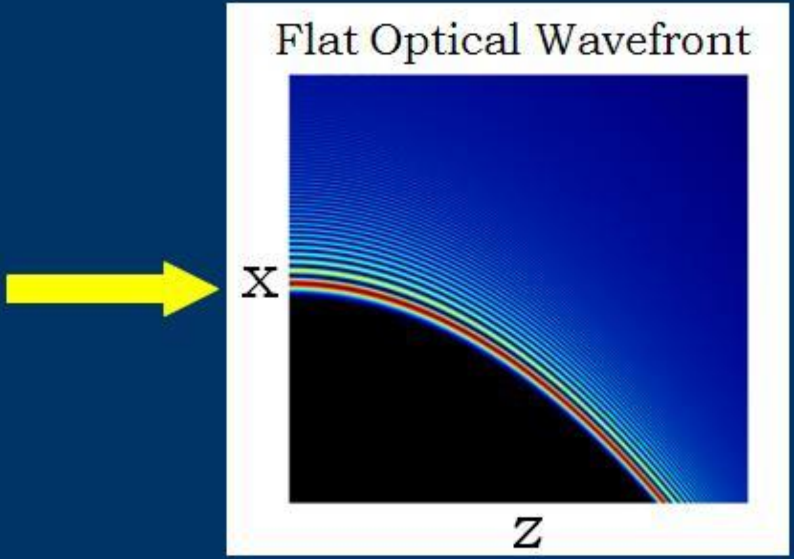
Nephroid caustics

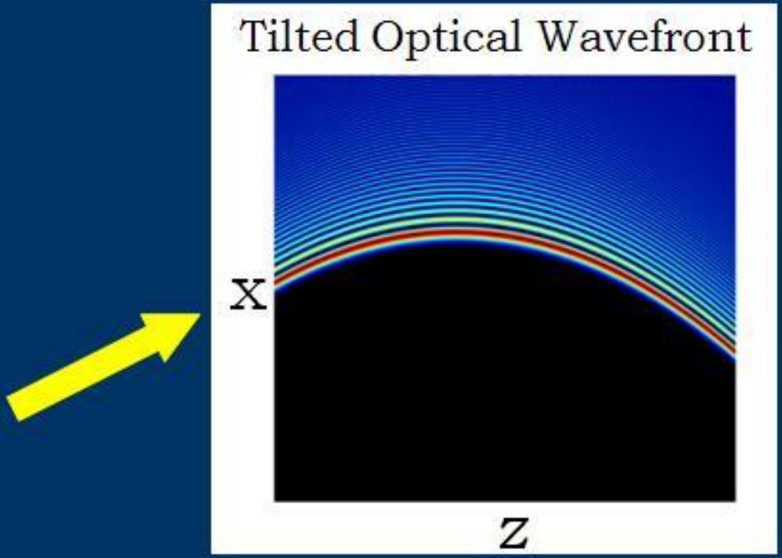
Still: Optical analog of projectile ballistics

The Airy beam moves on a parabolic trajectory very much like a projectile under the action of gravity!



It looks like
But it's not
What it looks like

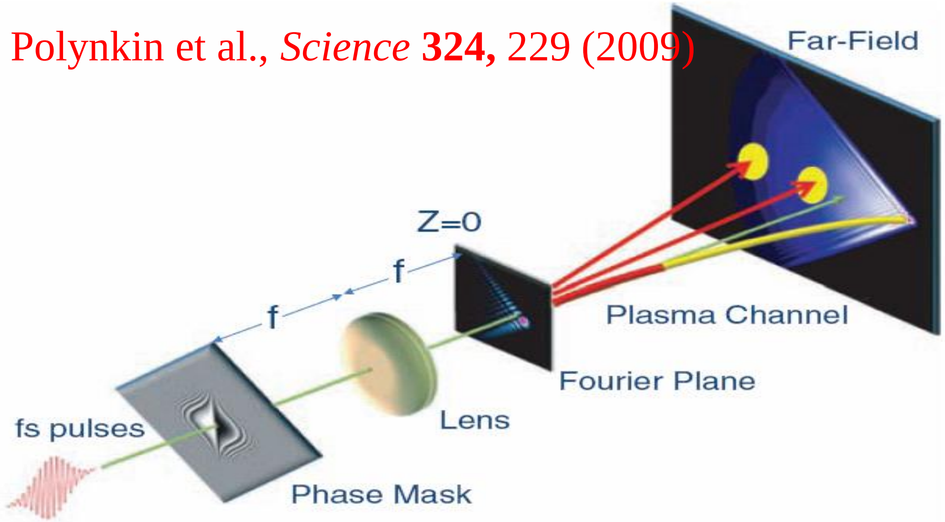




Applications of Airy beams

Curved plasma channel generation in air

Polynkin et al., *Science* **324,** 229 (2009)



Transporting micro-particles

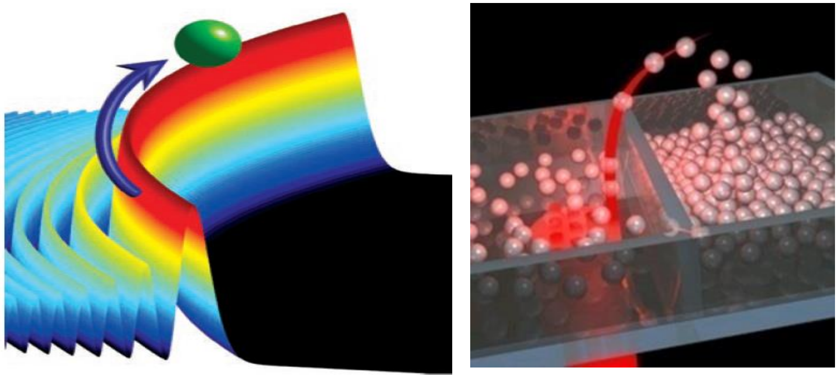
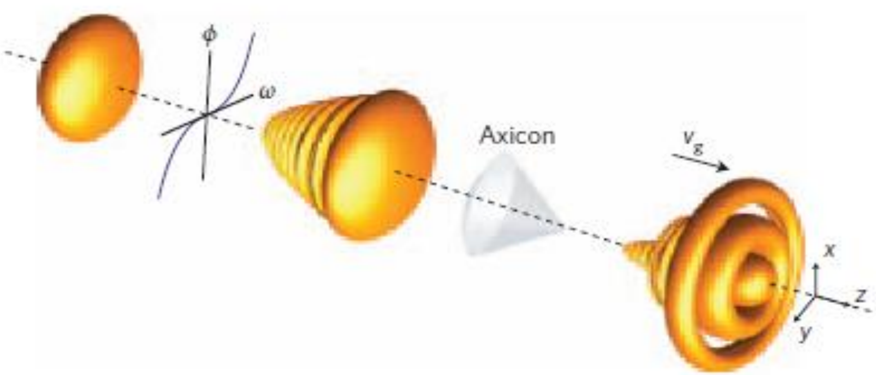


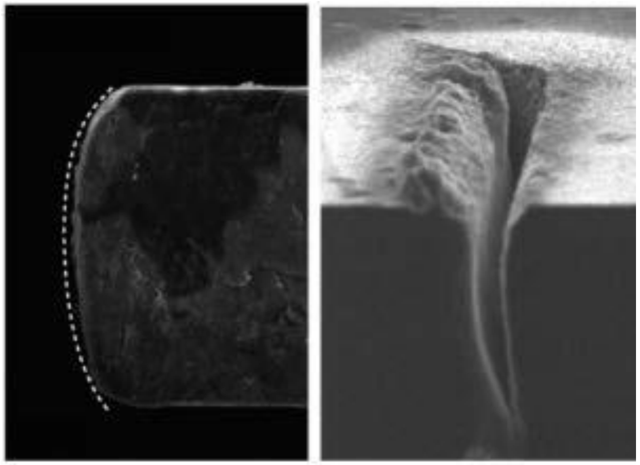
Figure 1 Propelling a microparticle (green sphere) along the curved path of an Airy beam.

Baumgartl, *Nature Photonics* **2,** 675 (2008)

Airy–Bessel wave packets as versatile linear light bullets in 3D



Chong et al, *Nature Photonics* **4,** 103 (2010)



Micromachining using accelerating beams.

Mathis et al. *Appl. Phys. Lett*. **101**, 071110 (2012)

Airy beams propagating in NL media

- Equation

$$i\frac{\partial\psi}{\partial z} - i\frac{z}{2}\frac{\partial\psi}{\partial x} + \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \delta n\psi = 0.$$

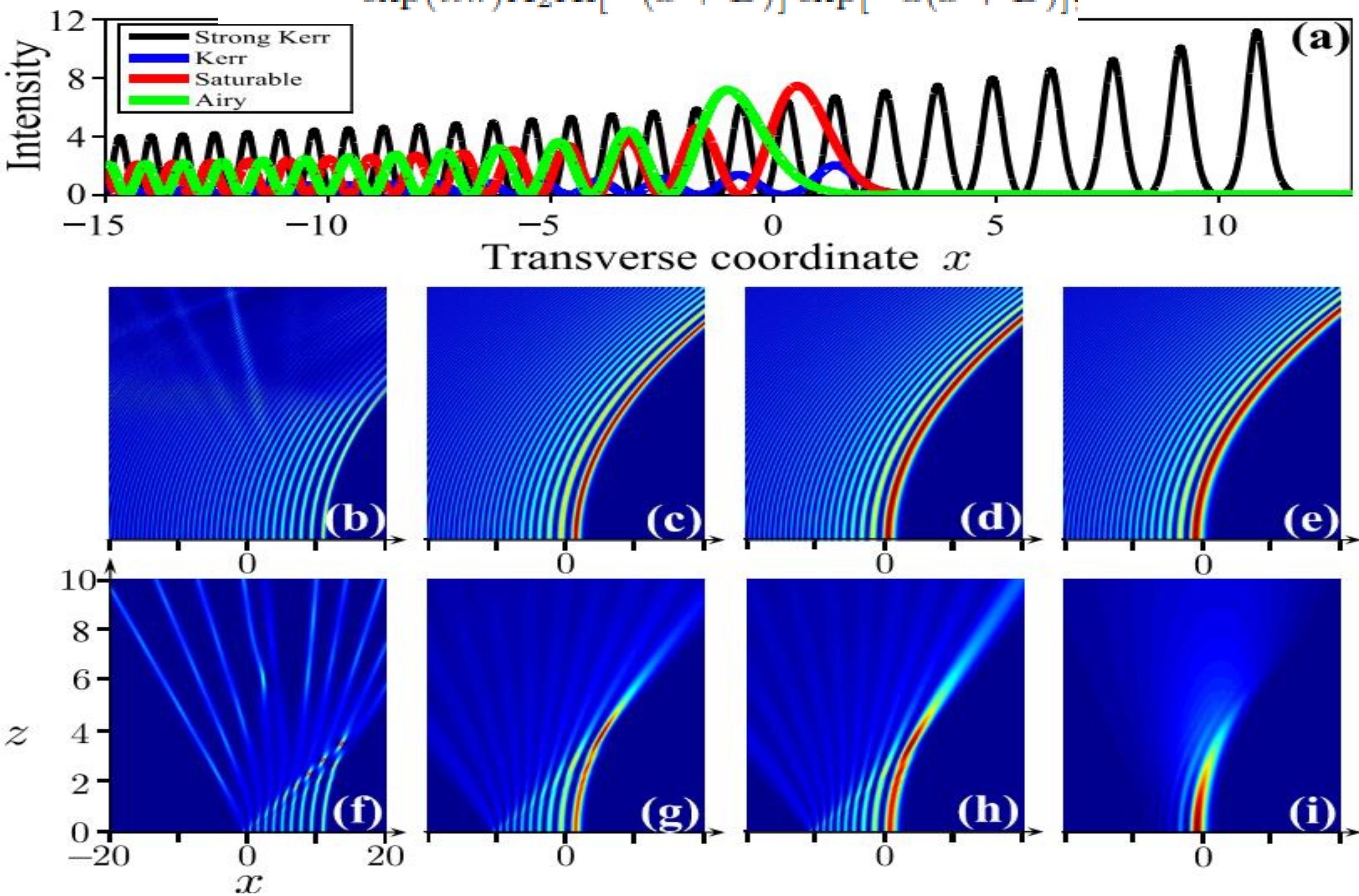
- Input beam

$$\psi(x) = A_1 \mathrm{Ai}[(x-B)]\exp[a(x-B)] + \exp(il\pi)A_2\mathrm{Ai}[-(x+B)]\exp[-a(x+B)].$$

Single

Beam

Prop



No trunca

tion, a=0

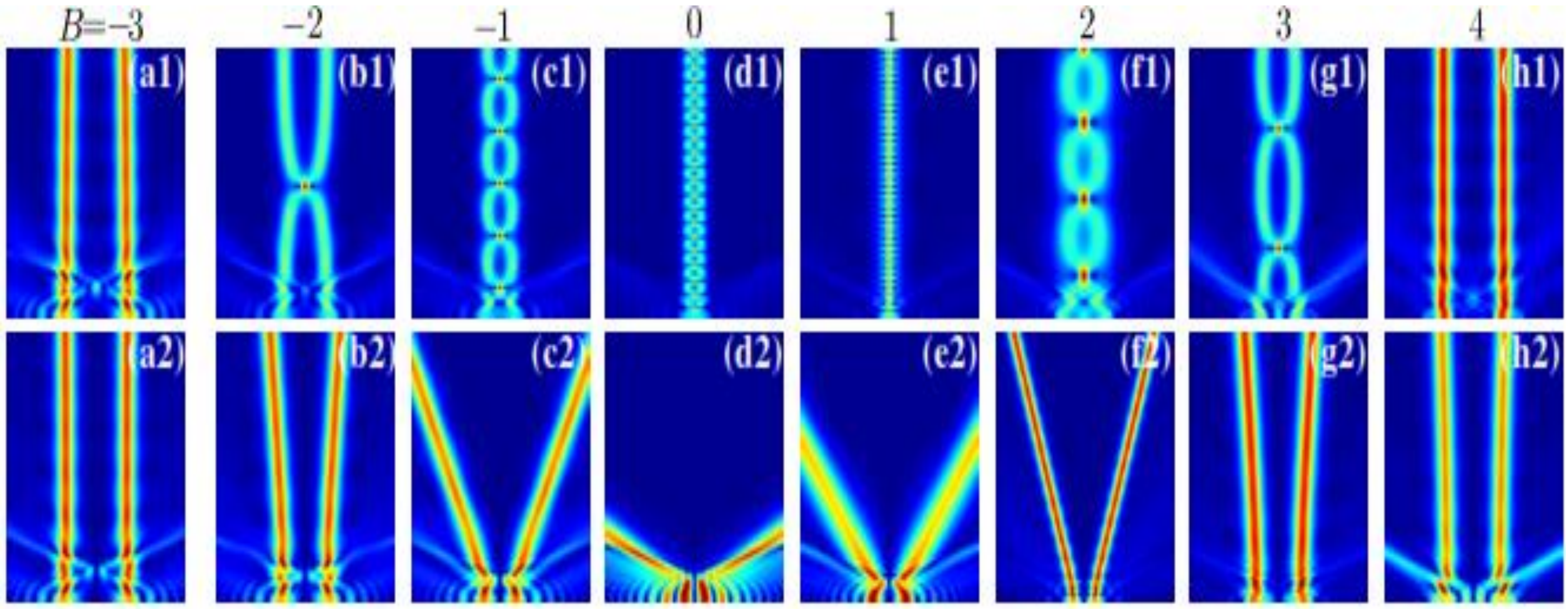
Truncated

a=0.2

Counter-accelerating NL Airy beams

- Kerr medium: Generation of solitons
- No acceleration!
- Upper row: In-phase (attraction)
- Lower row: Out-of-phase (repulsion)

Y. Zhang et al. *Appl. Sci*. 7, 341 (2017)



Paraxial accelerating beams; 2D

- Paraxial wave equation

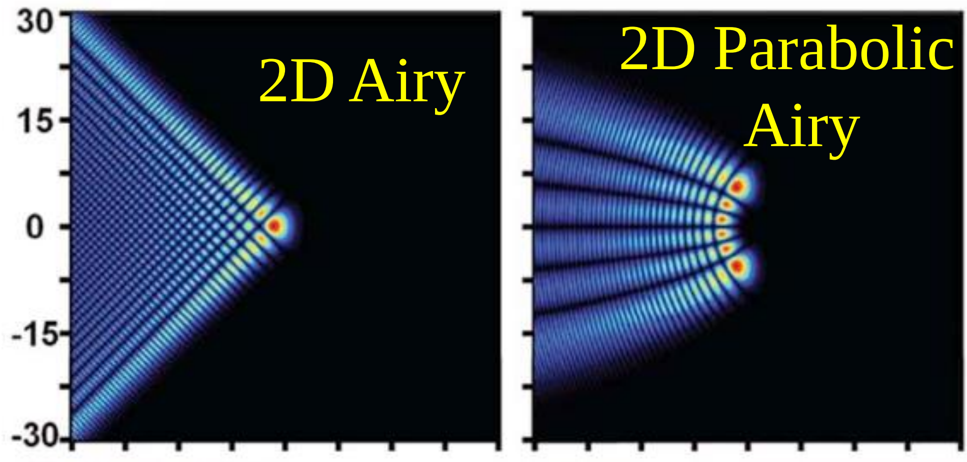
$$\nabla^2\psi + i\partial_s\psi = 0 \qquad s = z/2k\kappa^2$$

- Solution $\psi(u,v,s) = e^{-is\mathbf{p}^2}\psi(u,v,0) \qquad \mathbf{p}^2 = -\nabla^2$
- 1D case: Usual parabolic Airy beam
- 2D case: $e^{-is\mathbf{p}^2} = e^{-i2s^3/3}e^{isu}e^{-is^2p_u}e^{-is\hat{\mathrm{H}}}$, $\hat{\mathrm{H}} = -(\partial_u^2 + \partial_v^2) + u$, $p_u = -i\partial_u$
- Initial eigenvalue problem: $\hat{\mathrm{H}}\psi(u,v,0) = \lambda\psi(u,v,0)$

$$\psi_0(u,v) = \frac{1}{2\pi}\int e^{i\mathbf{k}\cdot\rho}\widetilde{F}_0(k_u,k_v)\mathrm{d}\mathbf{k}, \qquad (k_u^2 + k_v^2 + i\partial_{k_u})\widetilde{F}_0(k_u,k_v) = 0$$

$$\psi(u,v,s) = e^{is(u-\lambda-s^2)}e^{is^3/3} \text{ x}$$

$$F_0(u-\lambda-s^2,v)$$



M. Bandres: *OL* 34, 3791 (2009)

Nonparaxial accelerating beams; 3D

- Helmholtz equation $\left(\partial_{xx}+\partial_{yy}+\partial_{zz}+k^2\right)\psi=0$
- Solution $\psi(\boldsymbol{r})=\int A(\theta,\phi)\exp(ik\boldsymbol{r}\cdot\boldsymbol{u})\,\mathrm{d}\Omega,$
- *A(θ,φ)* Angular spectrum function

$$\boldsymbol{u}=(\sin\theta\sin\phi,\cos\theta,\sin\theta\cos\phi)\qquad \mathrm{d}\Omega=\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

- Assume $A(\theta,\varphi)=g(\theta)\exp(im\varphi)$

$$\psi(\boldsymbol{r})=\int_0^{\pi}\int_{-\pi/2}^{\pi/2}g(\theta)\exp(im\phi)\exp(ik\boldsymbol{r}\cdot\boldsymbol{u})\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi,$$

- For different beams pick different spectral functions

Interesting cases with separable Helmholtz equation

Coordinate systems in which the Helmholtz equation is separable

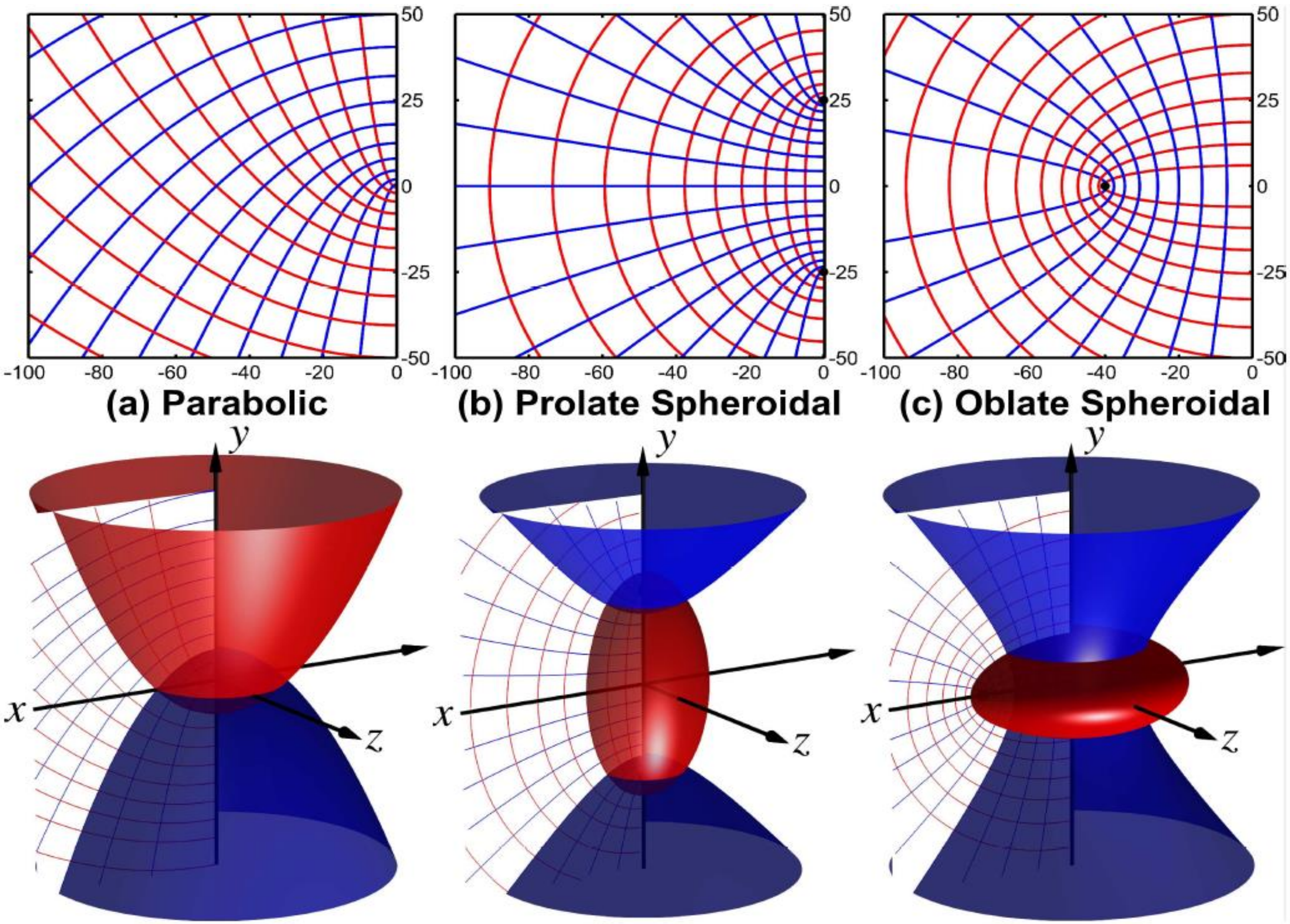
- Parabolic

acc beam (AB)

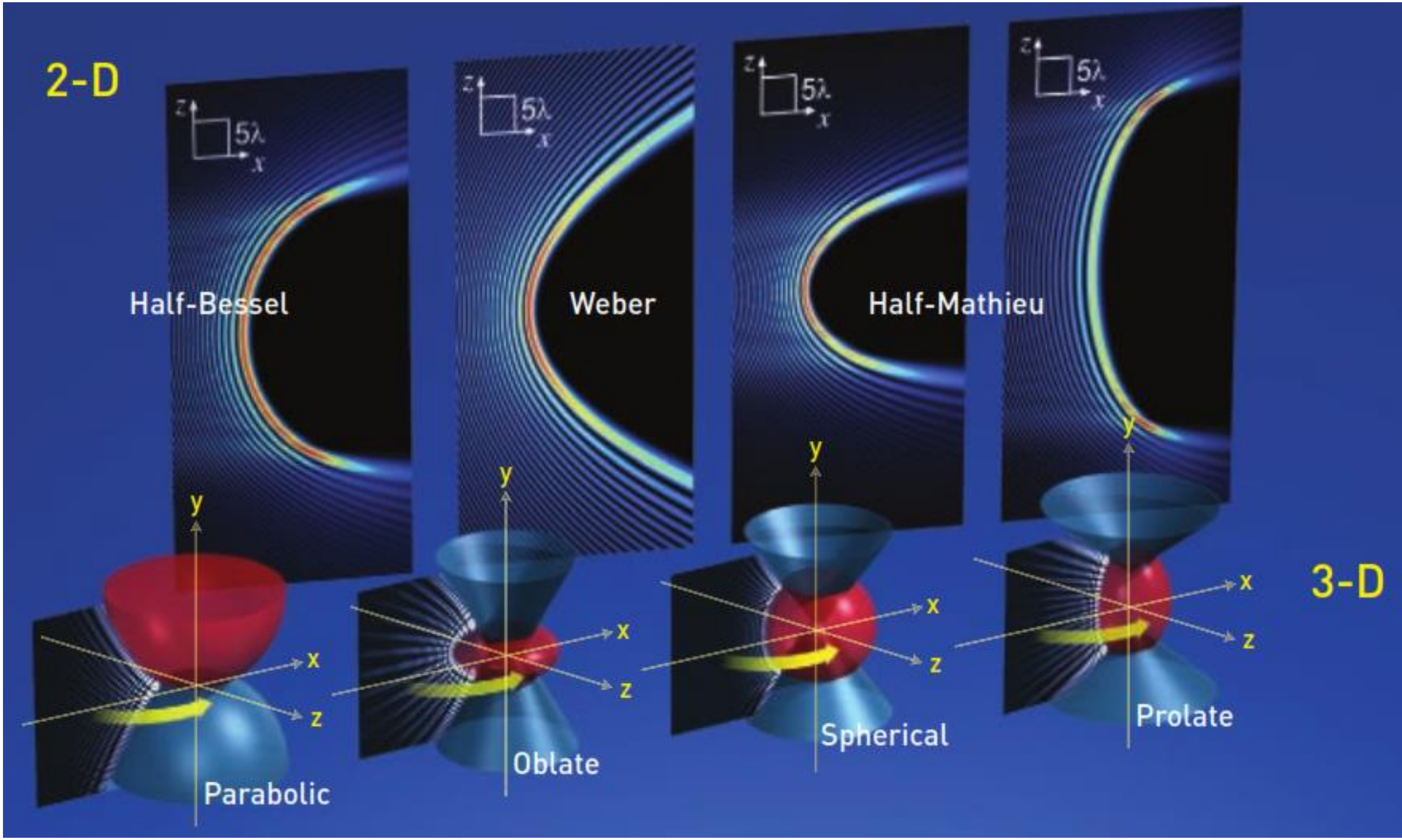
Spheroidal AB

- Oblate
- Prolate

- Spherical



Nonparaxial accelerating beams



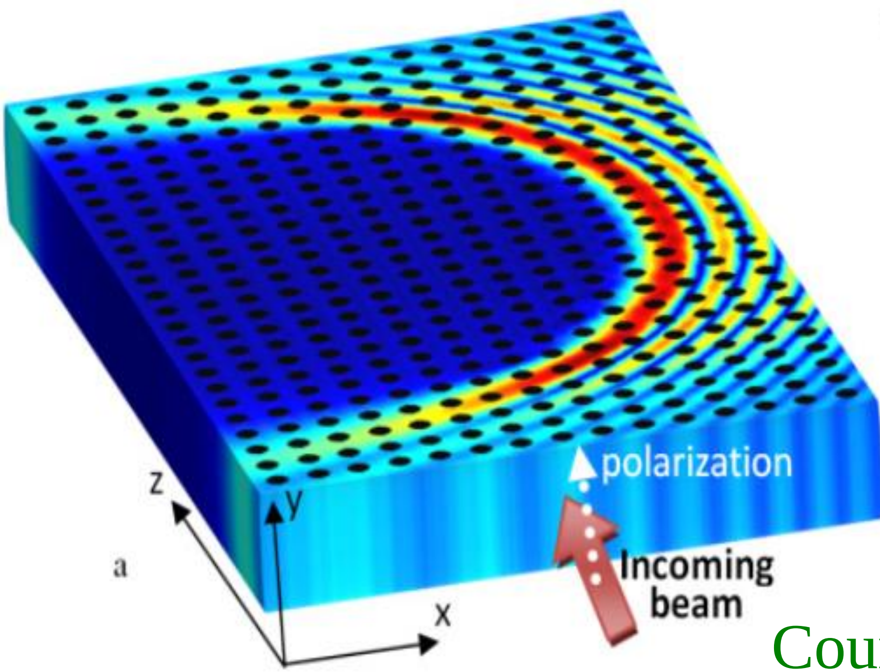
Accelerating beams in photonic crystals

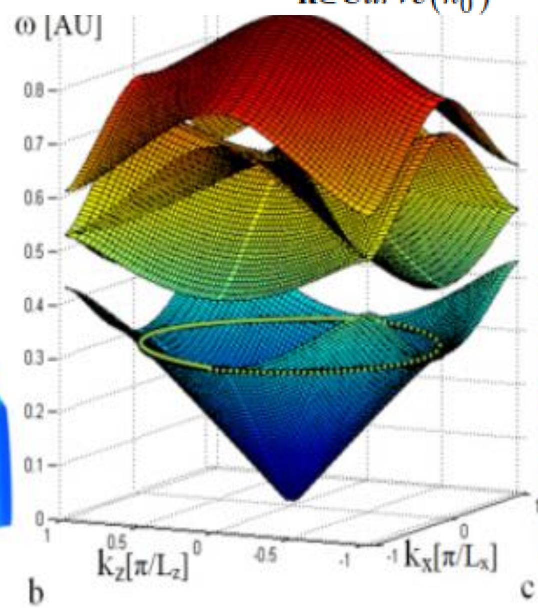
Beams in periodic systems: Energy band structure!

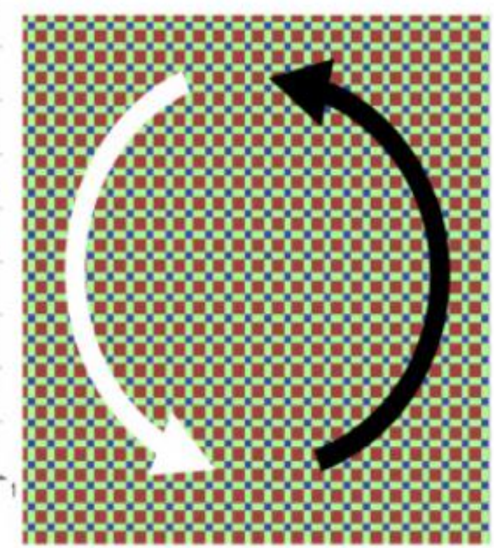
Highly nonparaxial: Helmholtz eq. $E_{xx}+E_{zz}+k_0^2\varepsilon(x,z)E=0$

Inherently counterprop: Forward and backward Bloch modes must be present: $u_{k_0,k_x,k_z}(x,z)e^{i\mathbf{k}\cdot\mathbf{r}}$ $\quad \mathbf{k}=(k_x,k_z)$

$$E_{k_0}((x,z)=\mathbf{r})=\oint_{\mathbf{k}\in Curve(k_0)} w(\mathbf{k})u_{k_0,\mathbf{k}}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}$$







Courtesy of M. Segev

Backward Forward

Accelerating beams in photonic crystals

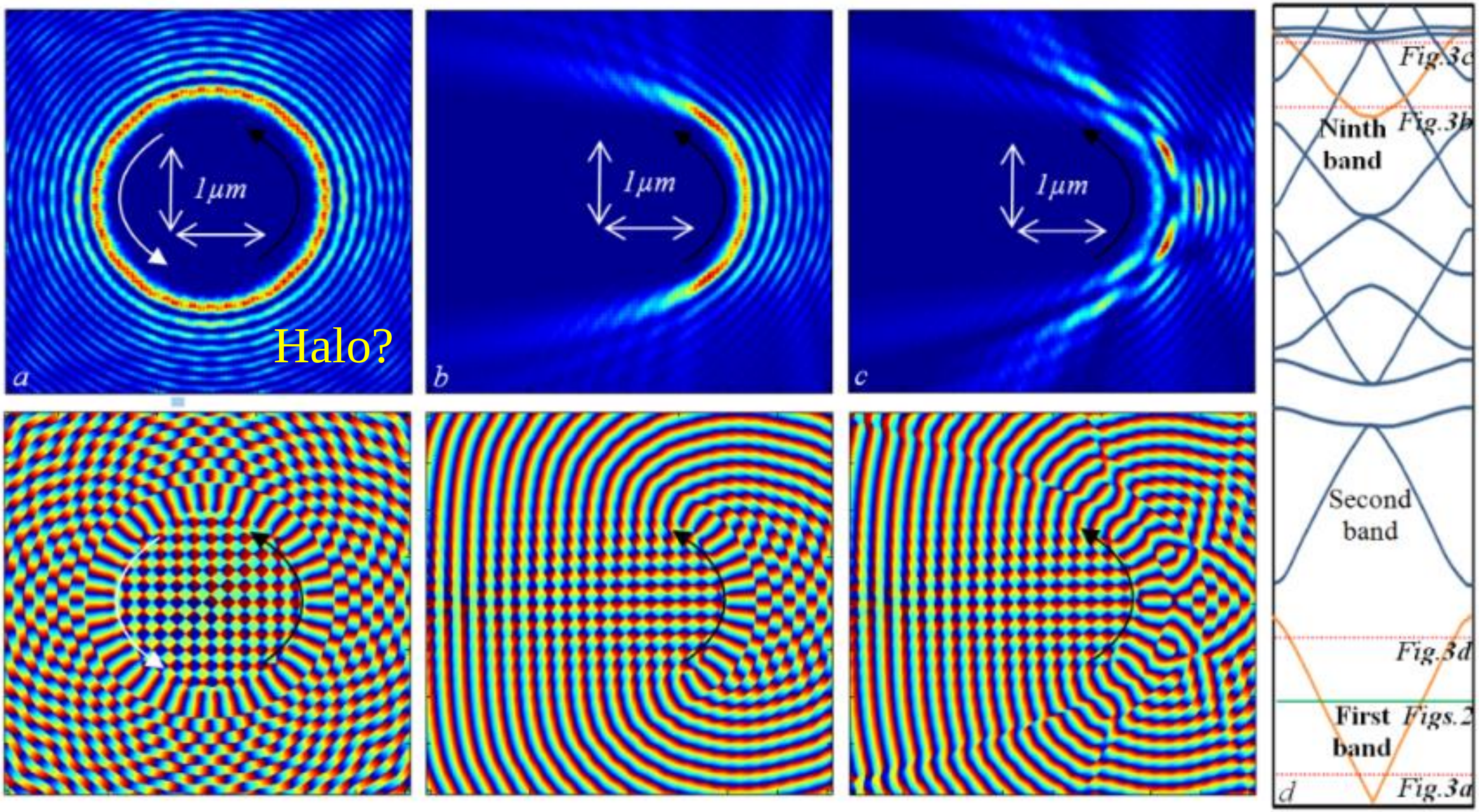


Fig. 2. Examples of self-accelerating beams in a 2D photonic crystal slab. For each solution, we plot the amplitude (top row) and phase (bottom row). The black arrows schematically mark the Poynting vector hence the energy flow. (a) The full "whirlpool beam", completing a full circle, is created from two counter-propagating input beams launched from top and bottom of

Dynamics of beams in Fractional SE

- Fascinating new field, “FQM” by Laskin, born in 21st century
- Longhi: “Fractional Schrödinger equation in optics”

$$i\frac{\partial F}{\partial t} + 1/2\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} F + V(x)\,F = 0$$

- NL:

$$i\frac{\partial F}{\partial z} + 1/2\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} F + |F|^2 F + V(x)\,F = 0$$

- The fractional Laplacian of order 1<α<2 is defined by:

$$\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} F(x) = \frac{1}{2\pi}\iint_{-\infty} \mathrm{d}p\mathrm{d}\xi |p|^{\alpha} F(\xi)\exp[ip(x-\xi)].$$

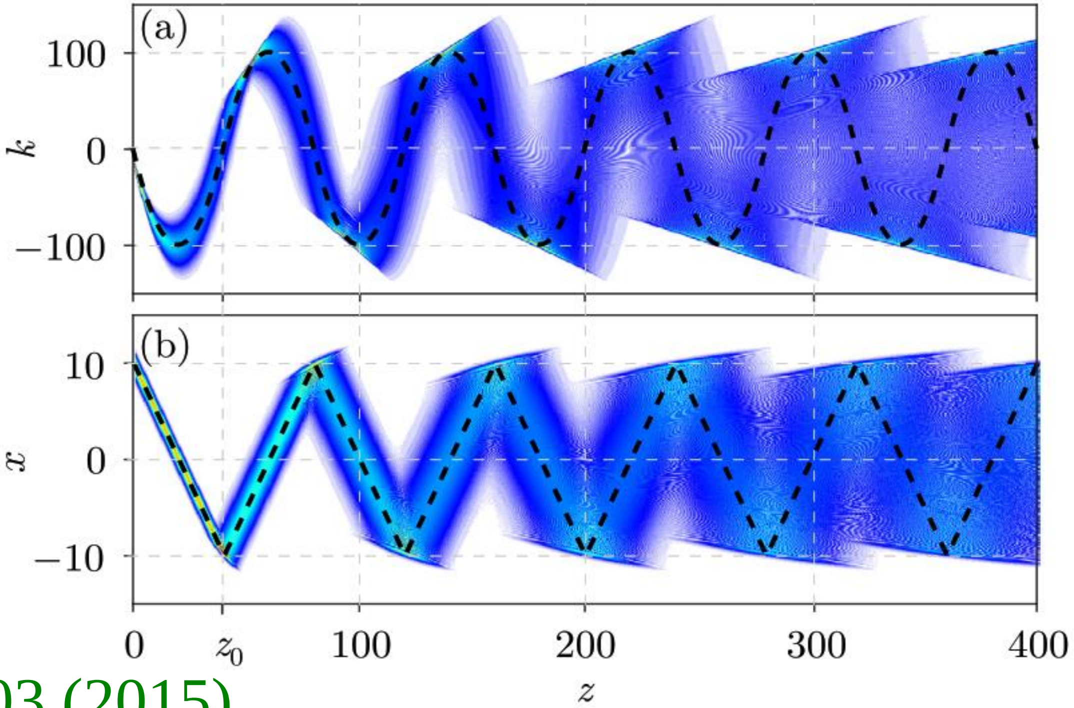
Go to inverse space, young man!

Representative results: Linear, α=1

- Harmonic potential $$i\frac{\partial F}{\partial t} + 1/2\left(-\frac{\partial^2}{\partial x^2}\right)^{1/2} F + x^2/2\, F = 0$$
- Equation in k-space $$i\frac{\partial \hat{\psi}}{\partial \xi} - \left(\frac{1}{2}|k| - \frac{1}{2}\frac{\partial^2}{\partial k^2}\right)\hat{\psi} = 0$$
- Input Gaussian beam $$\psi(x) = \exp\left[-\sigma(x - x_0)^2\right]$$
- Propagation of
- Displaced Gaussian

Dashed curves: Analytical

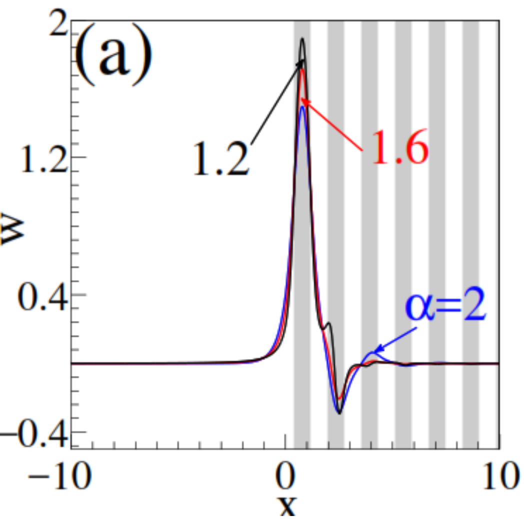
Shaded regions: Numerical



Y. Zhang *et al.*, *PRL* 115, 180403 (2015)

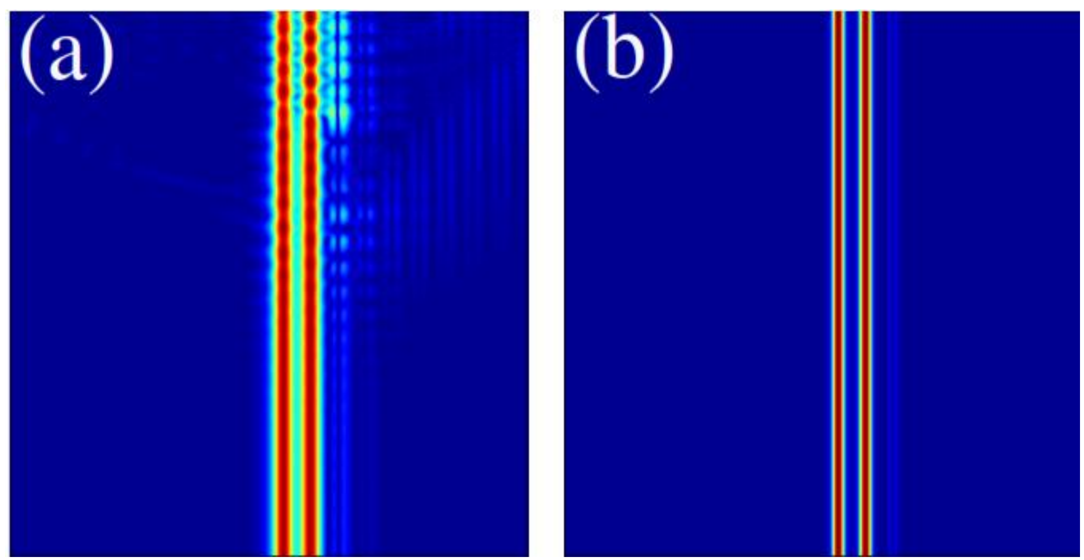
Representative results: Solitons in NL FSE

- Kerr nonlinearity $$i\frac{\partial F}{\partial z} + 1/2\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} F + |F|^2 F + R(x) F = 0$$
- Potential: Ref. index modulation in form of a lattice boundary $$R(x) = p[1-\cos(\Omega x)]$$
- Assumed solution $$F(x,z) = w(x)\exp(ibz)$$
- Equation $$\frac{1}{2}\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} w + |w|^2 w - Rw + bw = 0$$
- Formation of surface solitons at the boundary



Stable

Unstable→



Summary

Presented nondiffracting Airy wave-packets and beams

Airy (and other) wave-packets are freely accelerating and shape preserving

In optics it is a curving beam following parabolic trajectory

Presented various nonparaxial nondiffracting accelerating beams: Mathieu, Weber, Fresnel, Photonic, Fractional

Alluded on ways to introduce nonlinearities and applications

Thank you for your attention!