Graviton mass and Yukawa-like nonlinear correction to the gravitational potential: constraints from stellar orbits around the Galactic Center

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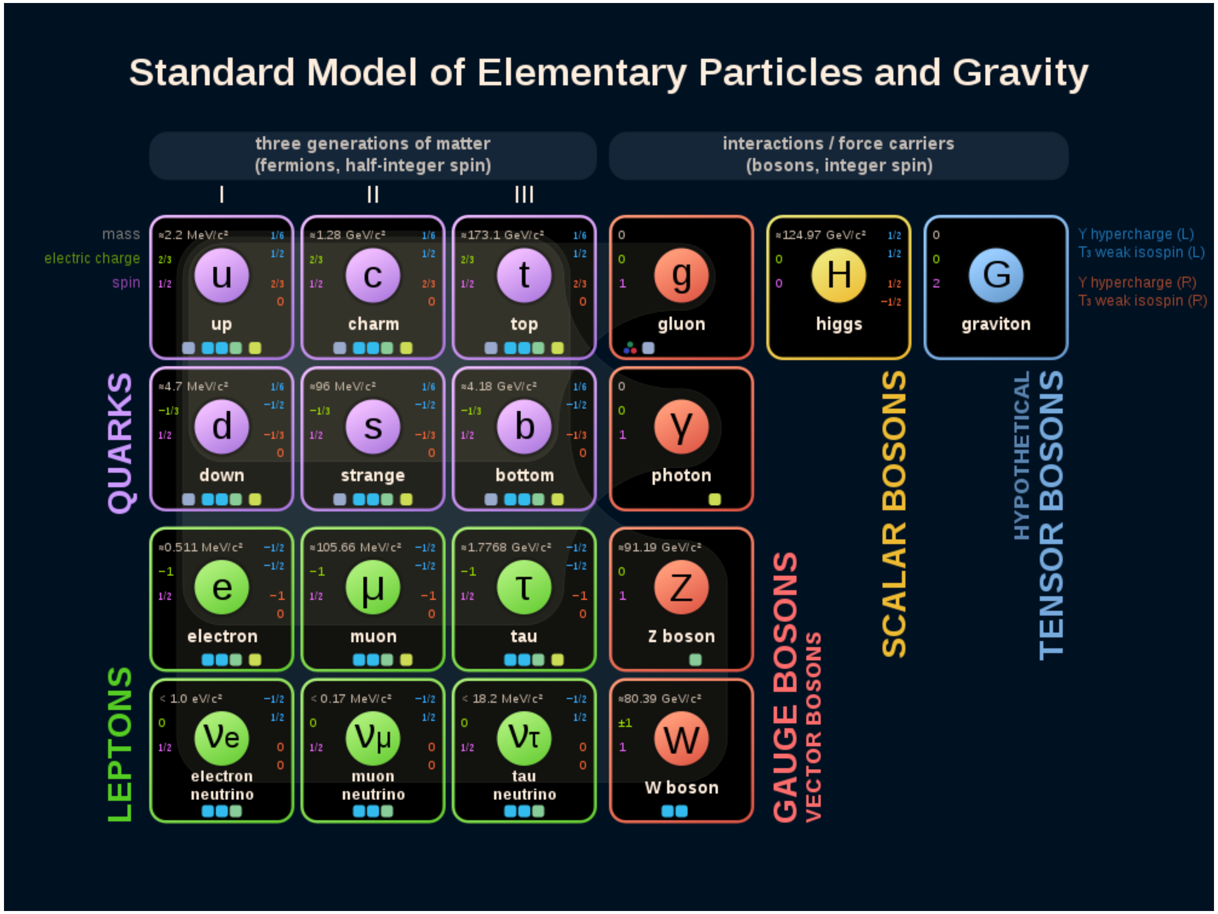
Outline of the talk

- Motivation: addressing the unclear situation with graviton mass
- Massive gravity and overview of the observational constraints on:
  - Compton wavelength and mass of graviton
  - Speed of gravity
- $f(R)$ gravity and Yukawa-like correction to the gravitational potential
- Observed stellar orbits around Sgr A* at Galactic Center
- Our results: constraining the graviton mass by analysis of the observed stellar orbits in Yukawa gravity
- The results are obtained in collaboration with:
  - Vesna Borka Jovanović and Duško Borka (Serbia)
  - Alexander F. Zakharov (Russia)
  - Salvatore Capozziello (Italy)
- Conclusions

Graviton: gauge boson of gravitational interaction

- Spin: 2 (tensor boson)
- Electric charge: 0 (neutral)
- General Relativity (GR): graviton is massless and travels along null geodesics (like photon), i.e. at the speed of light c
- Theories of massive gravity (indroduced by Fierz & Pauli, 1939, RSPSA, 173, 211): gravitation is propagated by a massive field (i.e. by graviton with small, nonzero mass m_g)
- Important predictions (Will, 1998, PRD, 57, 206):



1) The effective Newtonian potential has a Yukawa form: $\propto r^{-1}\exp(-r/\lambda_g)$, where $\lambda_g = h/(m_g\, c)$ is the Compton wavelength of graviton

2) Massive graviton propagates at an energy (or frequency) dependent speed

- Modified dispersion relation: $E^2 = p^2c^2 + m_g^2c^4 \quad \wedge \quad v_g^2/c^2 \equiv c^2p^2/E^2 \quad \Rightarrow$

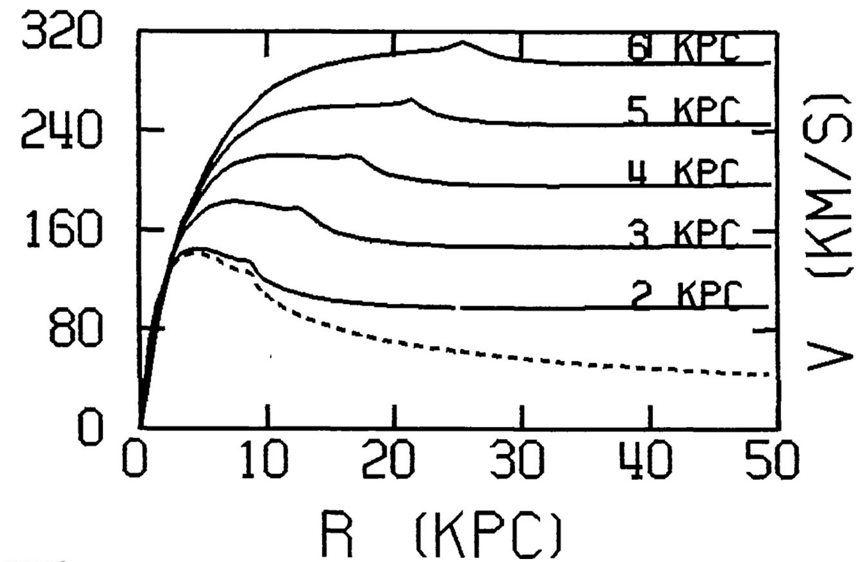
$$\boxed{v_g^2/c^2 = 1 - m_g^2c^4/E^2 = 1 - h^2c^2/(\lambda_g^2E^2) = 1 - c^2/(f\lambda_g)^2}$$

Constraints on Yukawa-like correction I

- Yukawa-like potential of the form (Sanders, 1984, A&A, 136, L21):

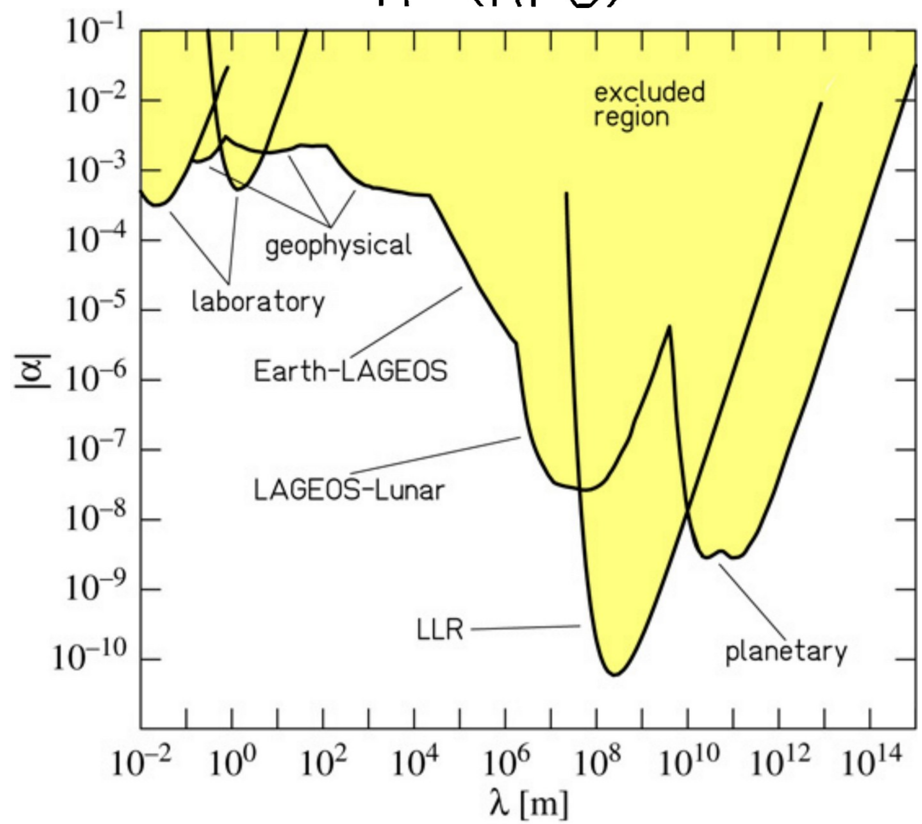
$$U(r) = \frac{G_\infty M}{r}\left(1 + \alpha\, e^{-r/r_0}\right), \quad r_0 = \frac{h}{m_0 c}$$

- Gravitational constant measured locally (G_0) and at infinity (G_∞): $G_0 = G_\infty(1+\alpha)$
- If r_0 corresponds to graviton mass m_0 then flat rotation curves could be accounted for $\alpha \sim -1$
- Additional repulsive (anti-gravity) force



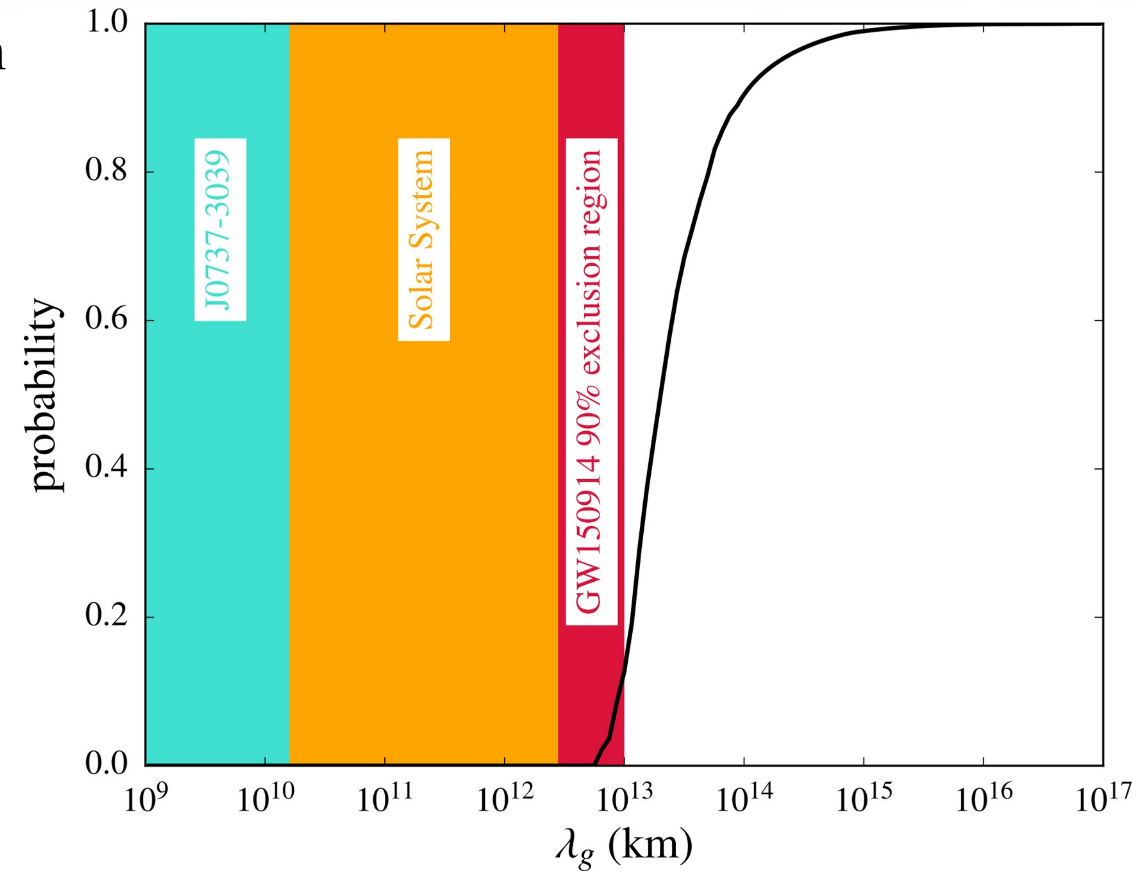
- Experimental constraints on additional Yukawa gravitational interaction between masses m_1 and m_2 (Adelberger et al. 2009, PrPNP, 62, 102):

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \alpha\, e^{-r/\lambda}\right)$$



Constraints on Yukawa-like correction II

- Probability distribution and exclusion regions for the graviton Compton wavelength λ_g (Abbott et al., LIGO Scientific and Virgo Collaborations, 2016, PRL, 116, 221101)



- Yukawa type correction with characteristic length scale λ_g:

$$\varphi(r) = \frac{GM}{r}\left(1 - e^{-r/\lambda_g}\right)$$

- LIGO bound from GW150914:

$$\boxed{\lambda_g > 1.6 \times 10^{13}\ \mathrm{km};\ m_g \leq 1.2 \times 10^{-22}\ \mathrm{eV}/c^2}$$

- Expected detection limit for a future pulsar timing array with 300 pulsars, observed for 10 years (Lee et al. 2010, ApJ, 722, 1589): $m_g = 5 \times 10^{-23}$ eV

- Weak lensing bounds (Rana et al. 2018, PLB, 781, 220): $\lambda_g > 6.82$ Mpc; $m_g < 6 \times 10^{-30}$ eV

Constraints on speed of gravity

- Constraints from the time difference Δt between a GW and an EW from the same event at a distance D (Will, 2014, LRR, 17, 4):

$$1 - \frac{v_{\rm g}}{c} = 5 \times 10^{-17} \left(\frac{200\ \rm Mpc}{D}\right)\left(\frac{\Delta t}{1\ \rm s}\right)$$

- Arrival and emission time differences $\Delta t_{\rm a}$ and $\Delta t_{\rm e}$: $\Delta t \equiv \Delta t_{\rm a} - (1+z)\Delta t_{\rm e}$

- For a massive graviton:

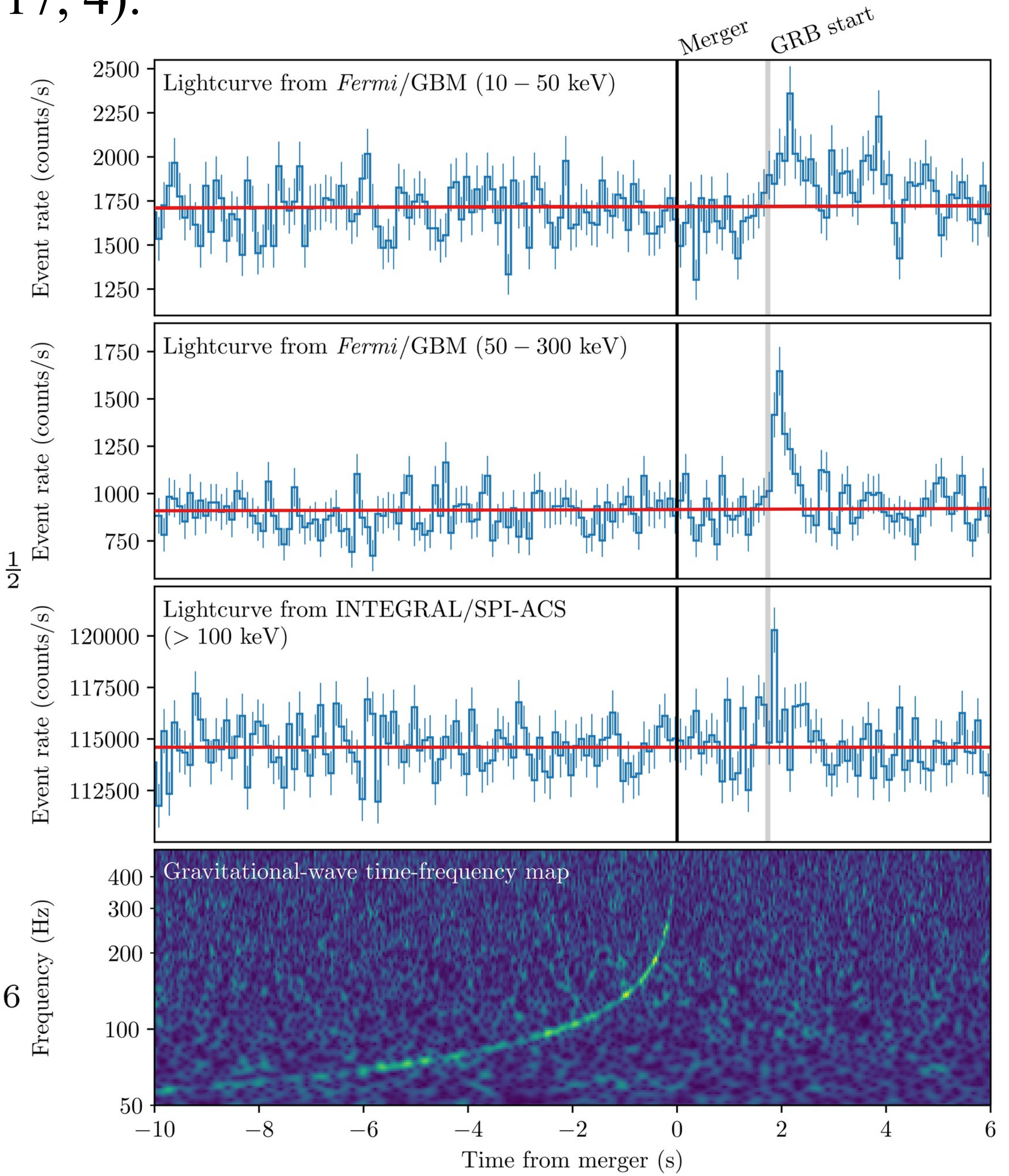
$$\frac{v_{\rm g}}{c} \approx 1 - \frac{c^2}{2(\lambda_{\rm g} f)^2} \ \Rightarrow$$

$$\lambda_{\rm g} > 3 \times 10^{12}\,{\rm km} \left(\frac{D}{200\,{\rm Mpc}}\frac{100\,{\rm Hz}}{f}\right)^{\frac{1}{2}} \left(\frac{1}{f\Delta t}\right)^{\frac{1}{2}}$$

- GW and γ-rays from a binary neutron star merger in the galaxy NGC 4993 at $z \approx 0.01$ and $D = 26$ Mpc (Abbott et al. 2017, ApJL, 848, L13):

$$\Delta t = \Delta t_{\rm a} = 1.74\,{\rm s} \ \Rightarrow \ \frac{v_{\rm g}}{c} - 1 \le +7 \times 10^{-16}$$

$$\Delta t = 10\,{\rm s} \ \Rightarrow \ \frac{v_{\rm g}}{c} - 1 \ge -3 \times 10^{-15}$$



$f(R)$ gravity and Yukawa-like nonlinear correction to the gravitational potential

- Gravitational potential with a Yukawa correction can be obtained in the Newtonian limit of any analytic $f(R)$ gravity model (Capozziello et al. 2014, PRD, 90, 044052)
- Action for $f(R)$ gravity: $\mathcal{S} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]\,, \quad \mathcal{X} = \dfrac{16\pi G}{c^4}$
- 4th-order field equations: $f'(R)R_{\mu\nu} - \dfrac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \dfrac{\mathcal{X}}{2}T_{\mu\nu}$
- Trace: $3\Box f'(R) + f'(R)R - 2f(R) = \dfrac{\mathcal{X}}{2}T$
- Analytic Taylor expandable function $f(R)$:

$$f(R) = \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}R^n = f_0 + f_1R + \frac{f_2}{2}R^2 + \ldots \quad \Rightarrow$$

- Metric: $ds^2 = \left[1 + \dfrac{2\Phi(r)}{c^2}\right]c^2dt^2 - \left[1 - \dfrac{2\Psi(r)}{c^2}\right]dr^2 - r^2d\Omega^2$
- **Yukawa-like nonlinear correction** to the grav. potential in the weak field limit:

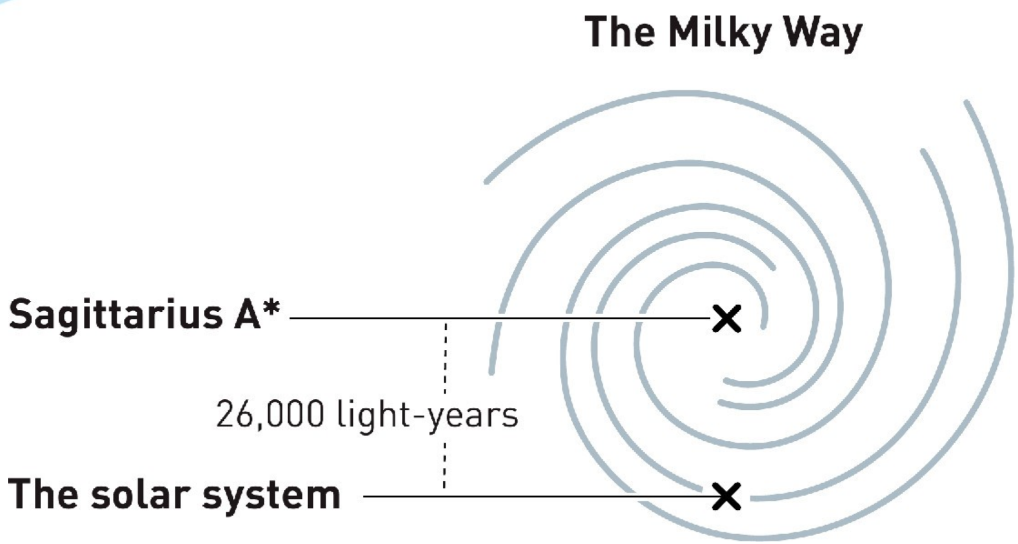
$$\boxed{\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right)}$$

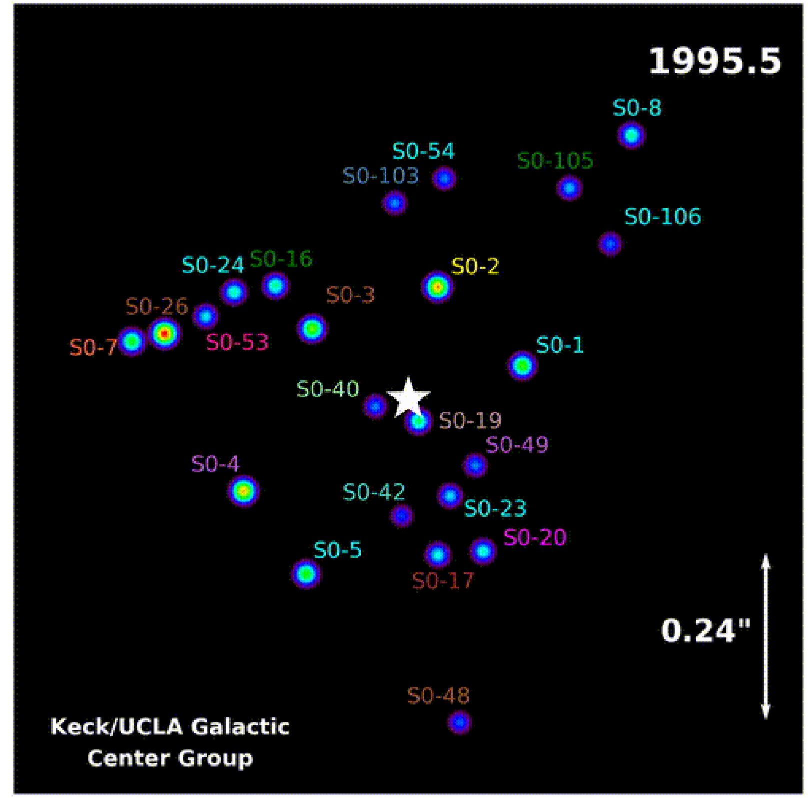
$$\Psi(r) = \frac{GM}{(1+\delta)r}\left[\left(1 + \frac{r}{\Lambda}\right)\delta e^{-\frac{r}{\Lambda}} - 1\right]$$

$$\Lambda^2 = -f_1/f_2 \quad \wedge \quad \delta = f_1 - 1$$

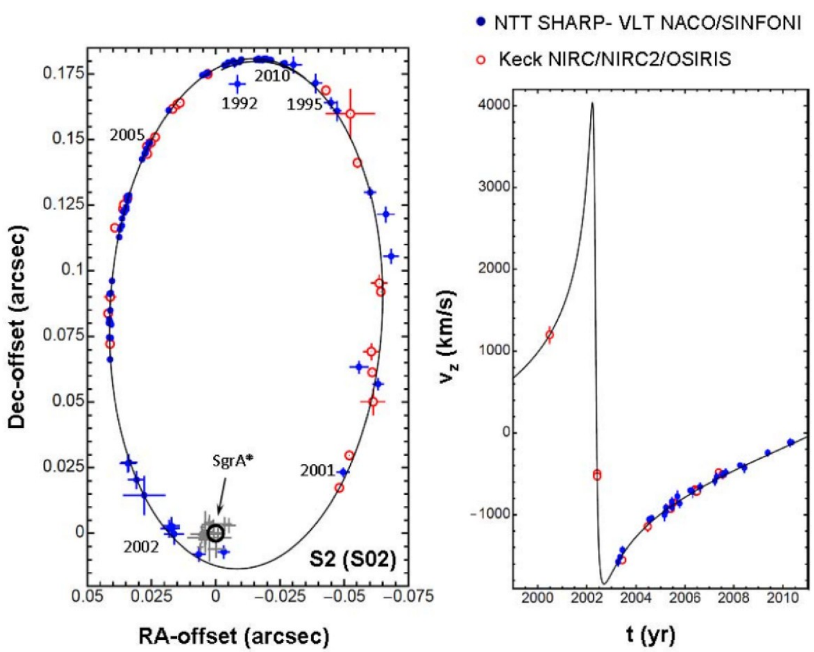
- **Λ - range of Yukawa interaction**
- **δ - universal constant**

Stellar orbits around Sgr A*

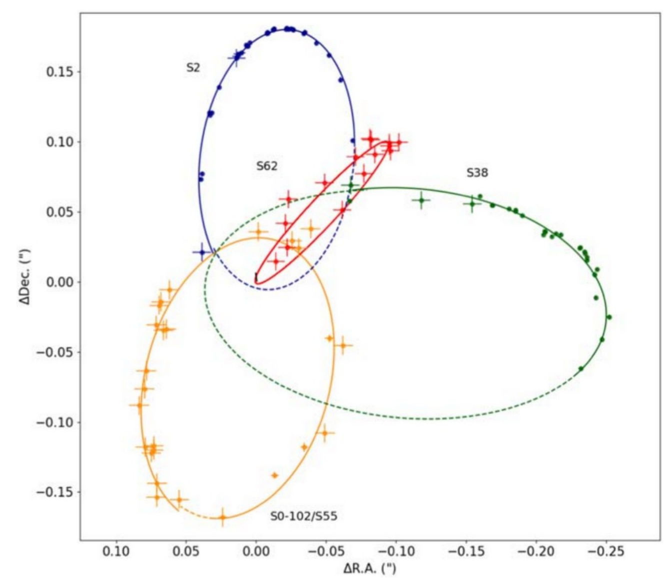




- Observations of S2 star by NTT/VLT and Keck:
  - Gillessen et al. 2009, ApJ, 692, 1075
  - Ghez et al. 2008, ApJ, 689, 1044



- New observations:
  - Gillessen et al. 2017, ApJ, 837, 30
  - GRAVITY, 2018, A&A, 615, L15
  - Peißker et al. 2020, ApJ, 889, 61 (right)

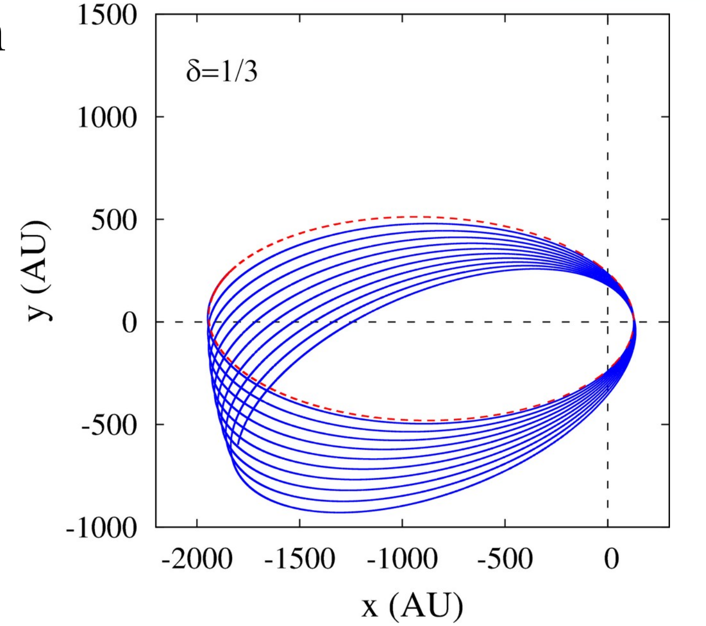


Simulated orbits of S2 star in Yukawa gravity

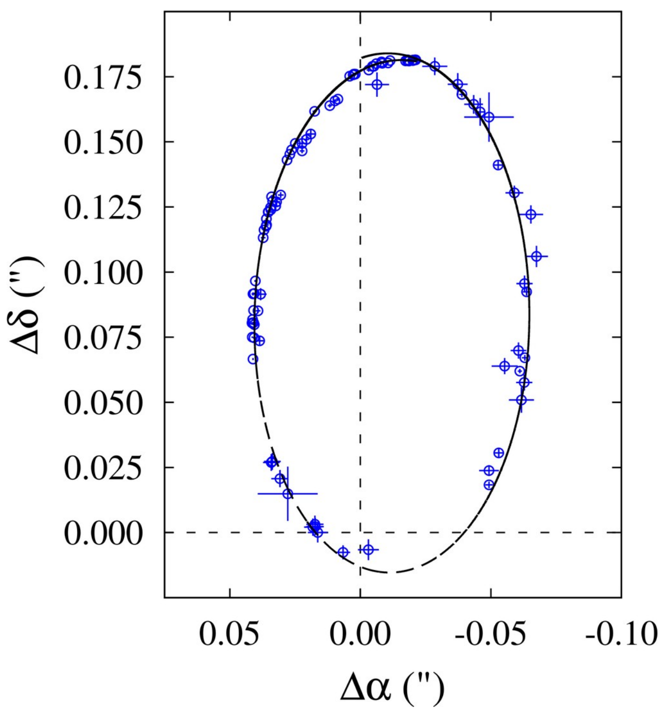
- Numerical integration of differential equations of motion (Borka, Jovanović, Borka Jovanović, Zakharov, 2013, JCAP, 2013, No. 11, 050): $\dot{\mathbf{r}} = \mathbf{v}, \quad \mu\ddot{\mathbf{r}} = -\nabla\Phi(\mathbf{r})$

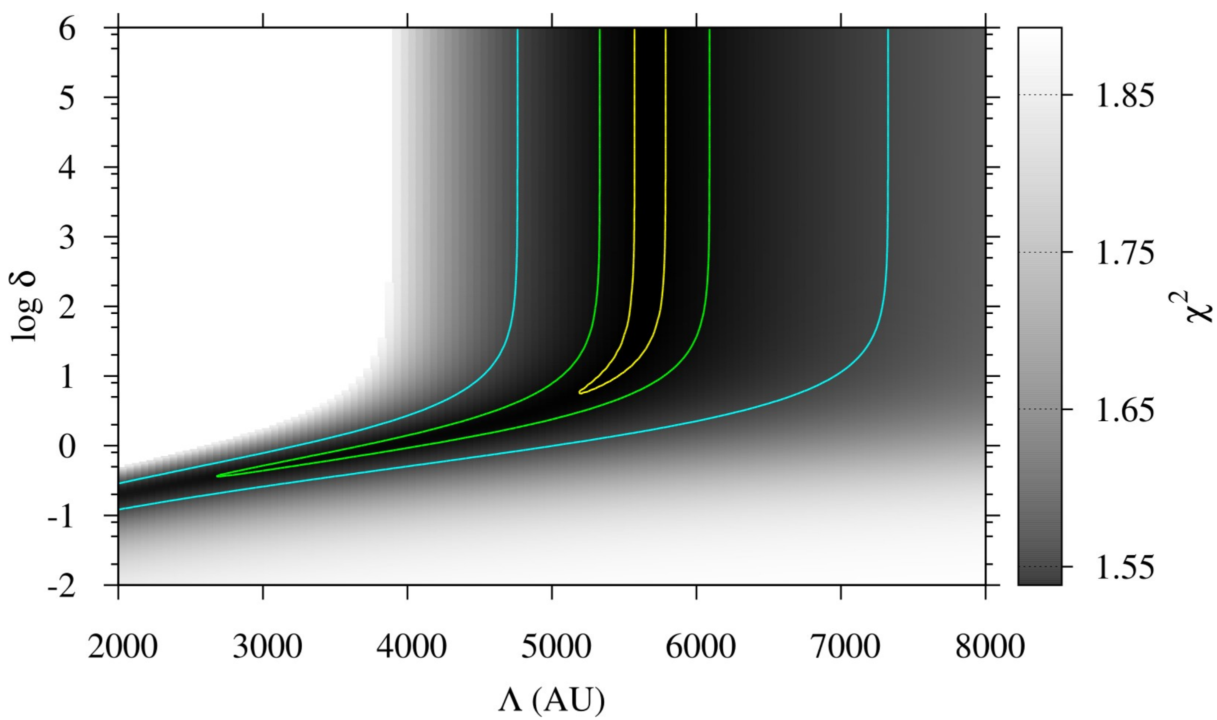
$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1+\delta e^{-\frac{r}{\Lambda}}\right) \Rightarrow \text{nonlinear equation}$$

of motion: $$\boxed{\ddot{\vec{r}} = -\frac{G(M+m)}{1+\delta}\left[1+\delta\left(1+\frac{r}{\Lambda}\right)e^{-\frac{r}{\Lambda}}\right]\frac{\vec{r}}{r^3}}$$



- Simulated orbits were then fitted to the astrometric observations of S2 star



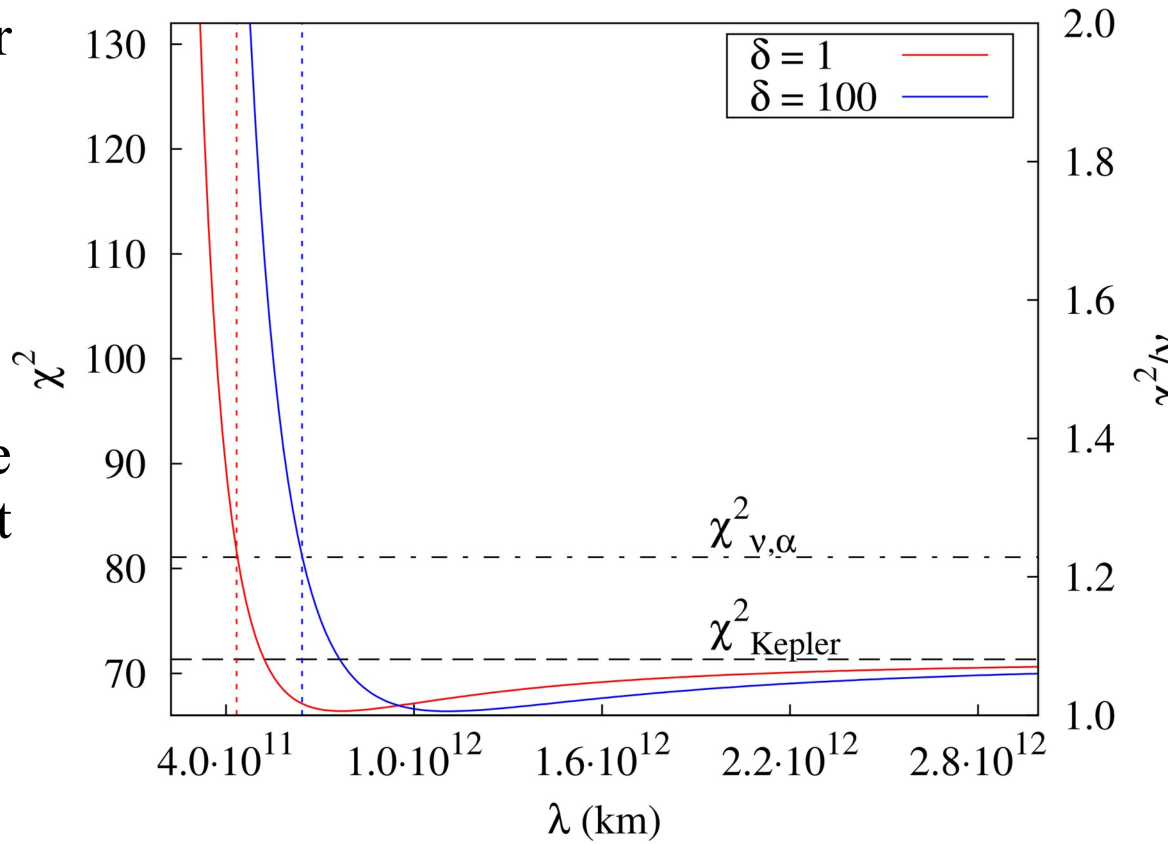


Our estimates for graviton mass upper bound

- χ^2 test of goodness of the S2 star orbit fits by Yukawa potential
- Test statistic:

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(x_i^o - x_i^c)^2}{\sigma_{xi}^2 + \sigma_{int}^2} + \frac{(y_i^o - y_i^c)^2}{\sigma_{yi}^2 + \sigma_{int}^2} \right]$$

- Intrinsic dispersion of the data due to their mutually inconsistent uncertainties: $\sigma_{int} = 1.13$ mas
- NDOF: $\nu = 66$
- Significance level: $\alpha = 0.1$
- Critical value for χ^2: $\chi^2_{\nu,\alpha} = 81.08$

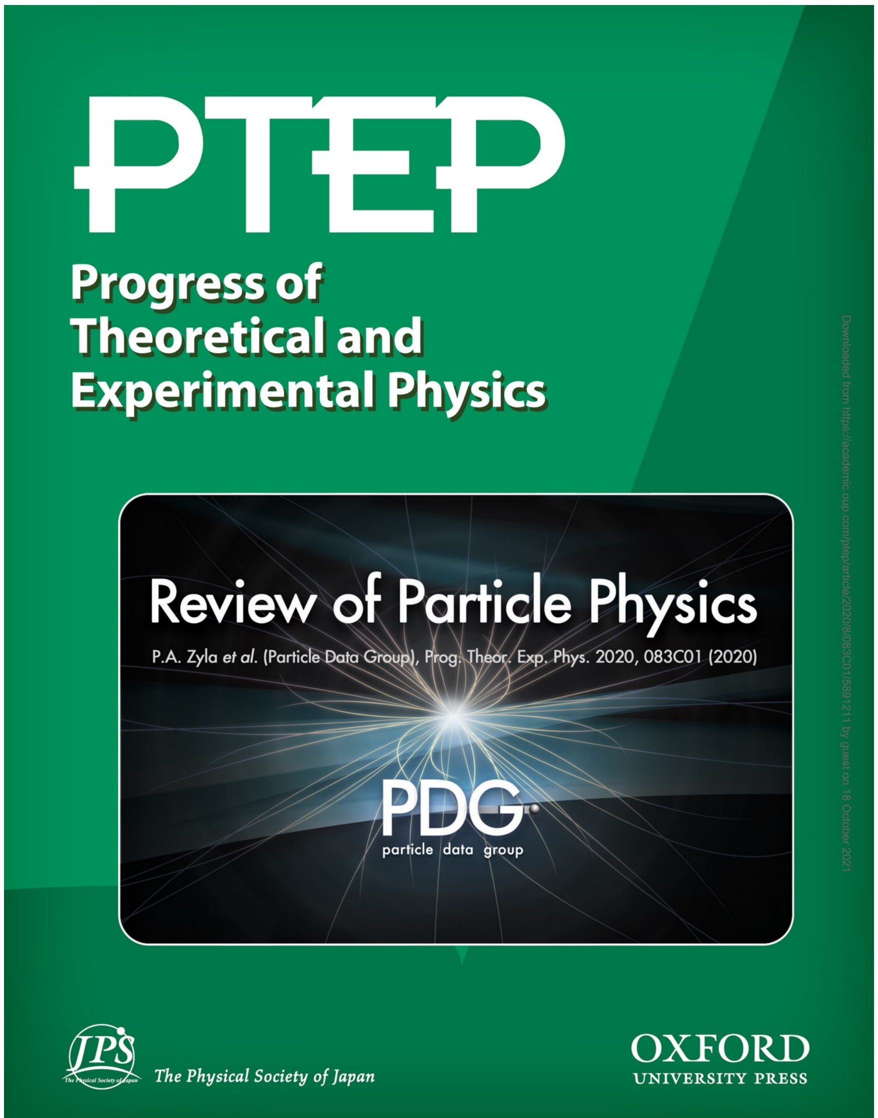


- Regions $\lambda < \lambda_x$ where $\chi^2 > \chi^2_{\nu,\alpha}$ can be excluded with 1 - α = 90% probability
- For $\delta = 1$: $\lambda_x = 2900 \pm 50\,\mathrm{AU} \approx 4.3 \times 10^{11}$ km
- For $\delta = 100$: $\lambda_x = 4300 \pm 50\,\mathrm{AU} \approx 6.4 \times 10^{11}$ km
- Corresponding upper bounds for graviton mass (Zakharov, Jovanović, Borka, Borka Jovanović, 2016, JCAP, 2016, No. 05, 045):

$$m_g = h\,c/\lambda_x \;\Rightarrow\; \boxed{m_g = 2.9 \times 10^{-21}\ \mathrm{eV} \quad \wedge \quad m_g = 1.9 \times 10^{-21}\ \mathrm{eV}}$$

Our estimates for graviton mass accepted by PDG

- From 2019, our estimate is in *Gauge and Higgs Boson Particle Listings* by PDG (Zyla et al., PDG, 2020, PTEP, 083C01)



Gauge & Higgs Boson Particle Listings

γ, g, graviton, W

graviton

$J = 2$

graviton MASS

VALUE (eV)	DOCUMENT ID		TECN	COMMENT
$<6 \times 10^{-32}$	[1] CHOUDHURY	04	YUKA	Weak gravitational lensing
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<6.8 \times 10^{-23}$	BERNUS	19	YUKA	Planetary ephemeris INPOP17b
$<1.4 \times 10^{-29}$	[2] DESAI	18	YUKA	Gal cluster Abell 1689
$<5 \times 10^{-30}$	[3] GUPTA	18	YUKA	SPT-SZ
$<3 \times 10^{-30}$	[3] GUPTA	18	YUKA	Planck all-sky SZ
$<1.3 \times 10^{-29}$	[3] GUPTA	18	YUKA	redMaPPer SDSS-DR8
$<6 \times 10^{-30}$	[4] RANA	18	YUKA	Weak lensing in massive clusters
$<8 \times 10^{-30}$	[5] RANA	18	YUKA	SZ effect in massive clusters
$<7 \times 10^{-23}$	[6] ABBOTT	17	DISP	Combined dispersion limit from three BH mergers
$<1.2 \times 10^{-22}$	[6] ABBOTT	16	DISP	Combined dispersion limit from two BH mergers
$<2.9 \times 10^{-21}$	[7] ZAKHAROV	16	YUKA	S2 star orbit
$<5 \times 10^{-23}$	[8] BRITO	13		Spinning black holes bounds
$<4 \times 10^{-25}$	[9] BASKARAN	08		Graviton phase velocity fluctuations
$<6 \times 10^{-32}$	[10] GRUZINOV	05	YUKA	Solar System observations
$<9.0 \times 10^{-34}$	[11] GERSHTEIN	04		From Ω_{tot} value assuming RTG
$>6 \times 10^{-34}$	[12] DVALI	03		Horizon scales
$<8 \times 10^{-20}$	[13,14] FINN	02	DISP	Binary pulsar orbital period decrease
	[14,15] DAMOUR	91		Binary pulsar PSR 1913+16
$<7 \times 10^{-23}$	TALMADGE	88	YUKA	Solar system planetary astrometric data
$< 2 \times 10^{-29}\ h_0^{-1}$	GOLDHABER	74		Rich clusters
$<7 \times 10^{-28}$	HARE	73		Galaxy
$<8 \times 10^{4}$	HARE	73		2γ decay

graviton REFERENCES

ABBOTT 16 PRL 116 061102 B.P. Abbott *et al.* (LIGO and Virgo Collabs.)
ZAKHAROV 16 JCAP 1605 045 A.F. Zakharov *et al.*
BRITO 13 PR D88 023514 R. Brito, V. Cardoso, P. Pani (LISB, MISS, HSCA+)

Possible improvements by future observations I

- Expected bounds for graviton mass if the GR predictions for orbital precession:

$$\Delta\varphi_{GR}^{rad} \approx \frac{6\pi GM}{c^2 a(1-e^2)}$$ will be confirmed by the future observations

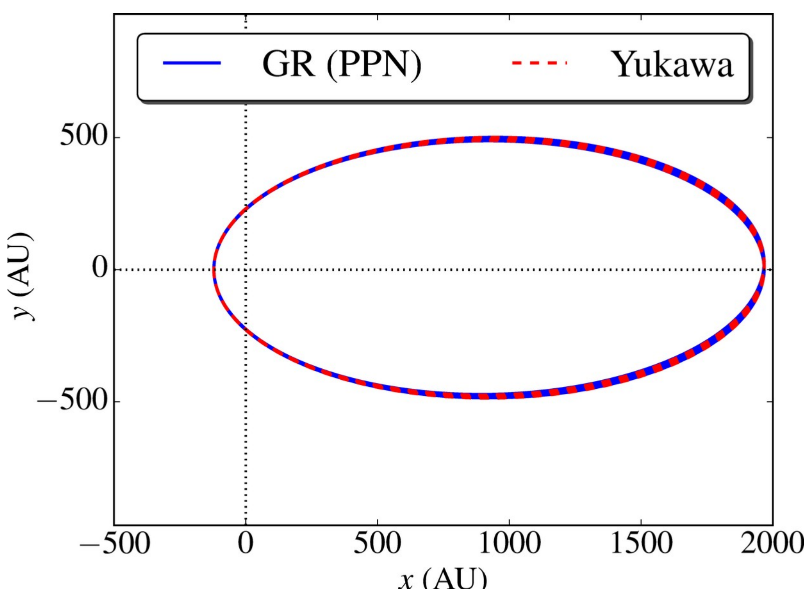
- Orbital precession in Yukawa gravity: $\Delta\varphi_{Y}^{rad} \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta}\frac{a^2}{\Lambda^2}, \quad a \ll \Lambda$

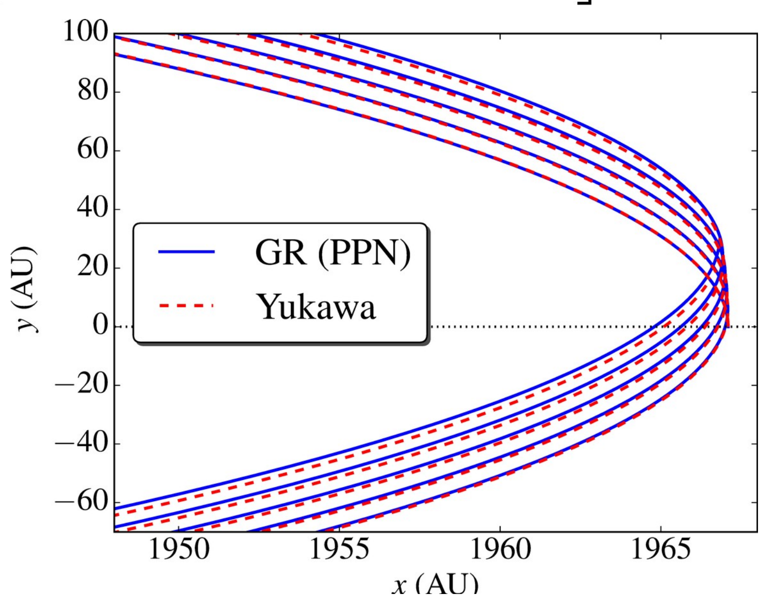
$$\Delta\varphi_Y = \Delta\varphi_{GR} \overset{\delta=1}{\Rightarrow} \boxed{\Lambda \approx \frac{c}{2}\sqrt{\frac{(a\sqrt{1-e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1-e^2})^3}{6R_S}} \approx \frac{T}{T_0}\sqrt{\frac{(a_0\sqrt{1-e^2})^3}{6R_S}},}$$

where T_0 and a_0 correspond to a selected orbit (e.g. of S2 star)

- Orbits in GR were simulated using PPN equation of motion for two-body problem:

$$\ddot{\vec{r}} = -G(M+m)\frac{\vec{r}}{r^3} + \frac{GM}{c^2 r^3}\left\{\left[2\left(\beta+\gamma\right)\frac{GM}{r} - \gamma\left(\dot{\vec{r}}\cdot\dot{\vec{r}}\right)\right]\vec{r} + 2\left(1+\gamma\right)\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right\}$$





(Zakharov, Jovanović, Borka, Borka Jovanović, 2018, JCAP, 2018, No. 04, 050)

Possible improvements by future observations II

- Relative errors: $\dfrac{\Delta\Lambda}{\Lambda} = \dfrac{\Delta m_g}{m_g} \approx \pm\dfrac{3}{2}\left(\dfrac{|\Delta a|}{a} + \dfrac{e\,|\Delta e|}{1-e^2} + \dfrac{1}{3}\dfrac{|\Delta M|}{M}\right)$

Star name	T_{Kep} (yr)	$\Delta\varphi$ ($''$)	Δs (mas)	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S1	168.4	48.2	0.22	369952.9 ± 43820.4	22.4 ± 2.7	11.8
S2	16.3	722.1	0.83	15125.5 ± 884.7	547.9 ± 32.0	5.8
S4	78.2	65.5	0.16	200418.3 ± 11191.1	41.4 ± 2.3	5.6
S6	195.5	102.4	0.60	226607.6 ± 8807.8	36.6 ± 1.4	3.9
S8	94.4	138.0	0.49	125957.9 ± 8420.6	65.8 ± 4.4	6.7
S9	52.2	124.3	0.27	101193.1 ± 9289.8	81.9 ± 7.5	9.2
S12	59.9	314.6	0.86	54047.1 ± 3026.3	153.3 ± 8.6	5.6
S13	49.8	91.6	0.17	124334.4 ± 5855.1	66.7 ± 3.1	4.7
S14	56.2	1465.9	4.02	16508.7 ± 2802.9	502.0 ± 85.2	17.0
S17	77.9	66.1	0.16	198588.4 ± 16771.2	41.7 ± 3.5	8.4
S18	42.6	107.1	0.18	102263.2 ± 5784.8	81.0 ± 4.6	5.7
S19	137.6	87.1	0.38	214568.1 ± 89676.6	38.6 ± 16.1	41.8
S21	37.6	217.4	0.41	56500.3 ± 4881.5	146.7 ± 12.7	8.6
S22	550.0	19.0	0.17	1347095.9 ± 580678.1	6.2 ± 2.7	43.1
S23	46.7	114.1	0.22	102079.9 ± 28448.6	81.2 ± 22.6	27.9
S24	336.5	107.5	0.93	286723.3 ± 41927.1	28.9 ± 4.2	14.6
S29	102.7	98.5	0.35	169074.0 ± 37807.8	49.0 ± 11.0	22.4
S31	110.4	63.3	0.21	244347.7 ± 17733.9	33.9 ± 2.5	7.3
S33	195.3	47.9	0.25	400731.7 ± 75407.3	20.7 ± 3.9	18.8
S38	19.5	427.5	0.53	24533.9 ± 1005.0	337.8 ± 13.8	4.1

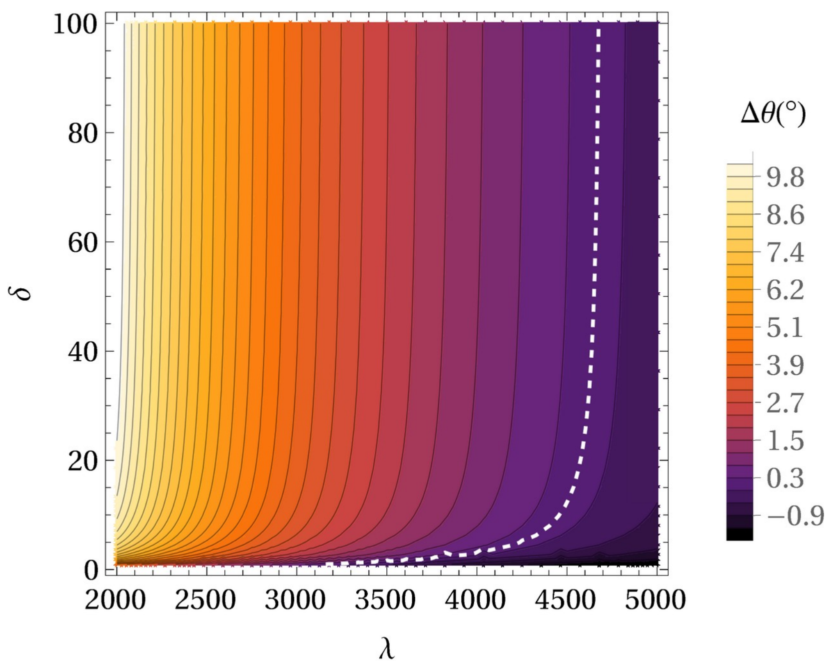
Star name	T_{Kep} (yr)	$\Delta\varphi$ ($''$)	Δs (mas)	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S39	82.6	364.5	1.26	56824.5 ± 6638.4	145.8 ± 17.0	11.7
S42	339.7	30.8	0.22	736342.1 ± 312551.0	11.3 ± 4.8	42.4
S54	482.2	81.5	0.90	422181.5 ± 692183.8	19.6 ± 32.2	164.0
S55	13.0	382.8	0.34	21721.6 ± 1465.5	381.5 ± 25.7	6.7
S60	88.6	105.5	0.34	149149.2 ± 11131.0	55.6 ± 4.1	7.5
S66	675.3	13.4	0.11	1933974.1 ± 269754.5	4.3 ± 0.6	13.9
S67	438.3	19.3	0.14	1188222.1 ± 116748.7	7.0 ± 0.7	9.8
S71	352.1	106.2	0.95	295890.2 ± 56006.2	28.0 ± 5.3	18.9
S83	667.2	15.3	0.15	1737943.9 ± 477698.2	4.8 ± 1.3	27.5
S85	3619.1	11.0	0.44	5195117.0 ± 8106789.5	1.6 ± 2.5	156.0
S87	1663.8	7.6	0.12	4641782.4 ± 619016.2	1.8 ± 0.2	13.3
S89	412.3	31.0	0.27	806547.1 ± 140413.3	10.3 ± 1.8	17.4
S91	973.6	11.4	0.14	2626592.1 ± 322729.8	3.2 ± 0.4	12.3
S96	673.2	13.6	0.12	1907722.2 ± 189196.9	4.3 ± 0.4	9.9
S97	1296.3	9.7	0.15	3407955.4 ± 1361276.1	2.4 ± 1.0	39.9
S145	434.8	23.6	0.19	1016130.7 ± 535791.9	8.2 ± 4.3	52.7
S175	97.7	1812.0	7.23	18570.1 ± 5168.9	446.3 ± 124.2	27.8
R34	893.3	18.6	0.27	1741793.7 ± 558192.6	4.8 ± 1.5	32.0
R44	2825.3	5.5	0.13	7740489.4 ± 5361256.0	1.1 ± 0.7	69.3

- Due to linear dependence of Λ on orbital period T, monitoring of a bright star with $T \approx 50$ yr and small eccentricity e could provide a constraint of: $m_g < 5 \times 10^{-23}$ eV
- In the case of S2 star, this estimate could reach: $m_g \approx 5.48 \times 10^{-22}$ eV
- Bounds from measured orbital precession of the Solar System planets (Will, 2018, CQG, 35, 17LT01): $m_g < 1 \times 10^{-23}$ eV

Influence of bulk distribution of matter

- Orbital precession of S2 star in Yukawa gravity for different mass densities of bulk distribution of matter which decribes stellar cluster, interstellar gas and dark matter, contained within some radius *r* around SMBH: $M(r) = M_{BH} + M_{ext}(r)$
- Double power-law mass density profile (where $r_0 = 10'' \ \wedge \ \alpha = 1.4$ for S2 star):

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}, \ \alpha = \begin{cases} 2.0 \pm 0.1, & r \geq r_0 \\ 1.4 \pm 0.1, & r < r_0 \end{cases} \ \Rightarrow \ M_{ext}(r) = \frac{4\pi \rho_0 r_0^{\alpha}}{3-\alpha} r^{3-\alpha}$$



- S2 star precession per orbital period in (λ, δ) parameter space for mass density of extended matter: $\rho_0 = 2 \times 10^8 \ M_{\odot}\mathrm{pc}^{-3}$
- White dashed line: the case when the precession is the same as in GR ($0^{\circ}.18$)

Table 1 The values of parameter λ (in AU) for different combinations of 3 values of parameter δ the 5 values of the mass density distribution of extended matter ρ_0

	ρ_0 (in $10^8 M_{\odot}\mathrm{pc}^{-3}$)				
	0	2	4	6	8
δ=1	15125	3130	2080	1597	1302
δ=10	20395	4425	3015	2370	1978
δ=100	21285	4640	3175	2500	2090

Table 2 The graviton mass (m_g) estimates corresponding to all mass density distributions presented in Table 1, in the case when Yukawa gravity parameter $\delta = 1$

ρ_0 (in $10^8 M_{\odot}\mathrm{pc}^{-3}$)	0	2	4	6	8
m_g (in 10^{-21} eV)	0.5	2.6	4.0	5.2	6.4

(Jovanović, Borka, Borka Jovanović, Zakharov, 2021, EPJD, 75, 145)

Conclusions

- Analysis of the stellar orbits around Sgr A* in the frame of a massive gravity with Yukawa-like nonlinear correction to the gravitational potential represents a powerful tool for constraining the graviton mass and testing the GR predictions
- Fitting the simulated stellar orbits in Yukawa potential to the observed orbit of S2 star showed that the range of Yukawa interaction Λ is on the order of several thousand astronomical units (AU)
- Assuming that Λ corresponds to the Compton wavelength of graviton λ_g, we estimated the upper bound for its mass to: $m_g \leq 2.9 \times 10^{-21}$ eV
- Our estimate for graviton mass upper bound is consistent with the LIGO results, but obtained in an independent way, and since 2019. it is included in the *Gauge and Higgs Boson Particle Listings* published by PDG
- Range of Yukawa gravity Λ can be constrained in such a way to induce the same orbital precession of stellar orbits as in GR
- Monitoring of a bright S-star with orbital period of $\approx$ 50 years and small eccentricity by the future telescopes could provide an opportunity to constrain the upper bound for graviton mass to: $m_g < 5 \times 10^{-23}$ eV
- Estimates for graviton mass may be slightly larger for larger mass density distributions of the extended matter, but are still in the expected interval

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Thank you for your attention!