Modeling dynamics of magnetization reversal in geometrically-frustrated nanonetworks

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Complexity at the Interplay of Topology and Dynamics

- Geometry: Higher-Order Simplexes (triangles, tetrahedrons, …) organized into Simplicial Complexes;

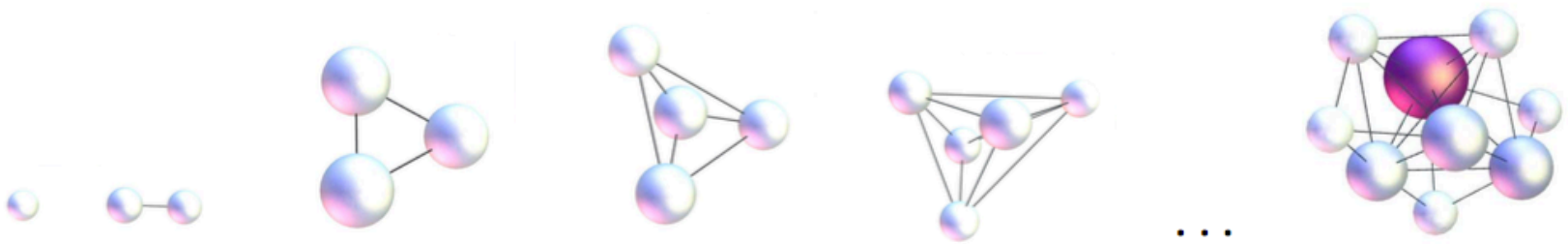
- Dynamics: Self-Organized Criticality (Collective Dynamic Phenomena in nonlinear interacting systems out of equilibrium) exhibiting the transition from elementary-to-global scale;

Contents

- Nanonetworks: Self-Assembly of Simplexes as a model for growing complex materials inspired by mathematics; {Materials Sci.}
- Spin kinetics on Simplicial Complexes: Field driven magnetization reversal & hysteresis loop phenomena; {Physics}
- Self-Organized Criticality at the Hysteresis Loop: Interplay of complex geometry and driven nonlinear dynamics; {Complexity}

Hidden geometry descriptors

Decomposition of graph structure into Simplexes; Cliques of different orders $q = 1, 2, 3, 4, 5, \cdots$



Uncovers hierarchical organisation of Simplicial Complexes occurring in the network; Reversed process: Composing networks with S.C. [*]

[*M. Šuvakov, M. Andjelković, B. Tadić, *Hidden geometry of networks arising from cooperative self-assembly*, Sci.Rep. 8:1987 (2018)]

Self-Assembly of Simplexes

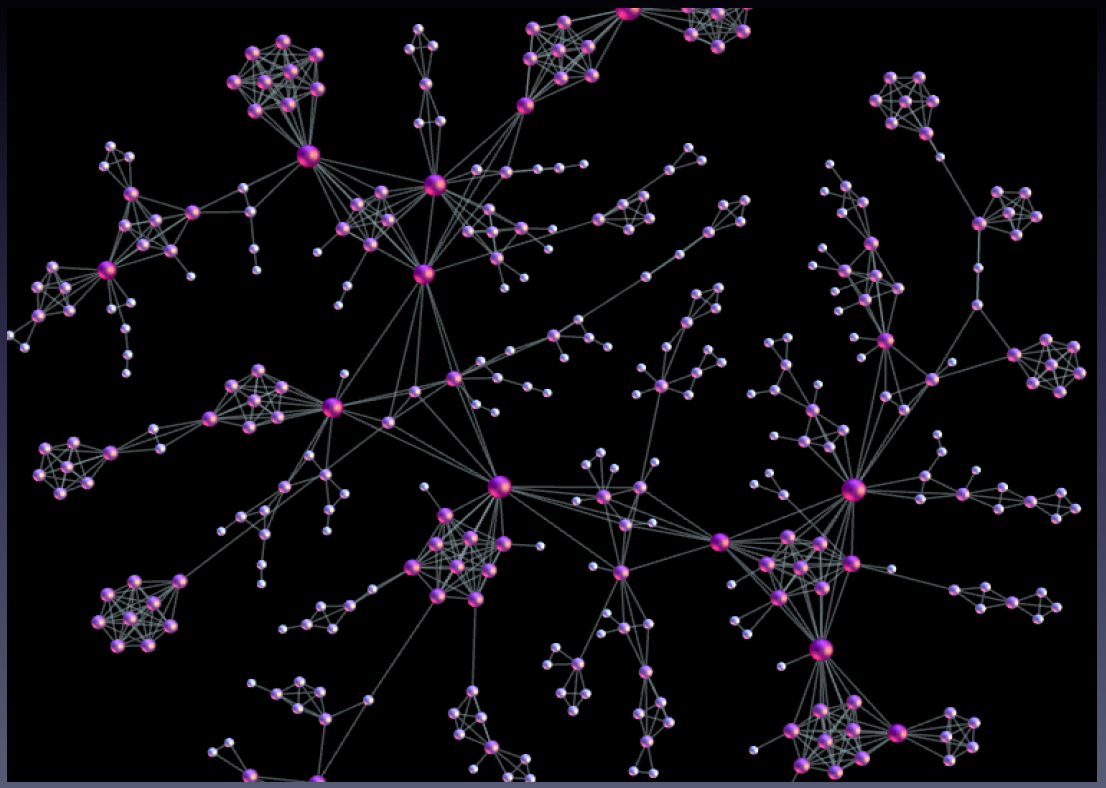
- Generative model of Simplicial Complexes (controlling the Higher order connectivity by self-assembly processes & parameters (n,ν): the size $n=q_{max}+1$ and geometrical compatibility of simplexes, chemical affinity, number of simplexes, defect bonds)

Suvakov, Andjelkovic, Tadic:
Hidden geometries in networks arising from Cooperative self-assembly,
Scientific Reports 8:1987 (2018)

Dankulov, Tadic, Melnik:
Spectral properties of hyperbolic nanonetworks with tunable aggregation of simplexes,
Phys. Rev. E 100(1): 012309 (2019)

Tadic, Andjelkovic, Suvakov, Rodgers,
Lagre-scale influence of defect bonds in geometrically-constrained self-assembly,
Phys. Rev. E 102(3): 032307 (2020)

$$p(q_{\max}, q; t) = \frac{c_q(t)e^{-\nu(q_{\max}-q)}}{\sum_{q=0}^{q_{\max}-1} c_q(t)e^{-\nu(q_{\max}-q)}}$$



Spin Reversal Dynamics on SC

- Hamiltonian with higher-order interactions embedded on the simplicial complexes:

$$\mathcal{H} = -\sum_{k'=2}^{k} \sum_{(i_1,\cdots i_{k'})\in\mathcal{F}_{k'}} J_{i_1,i_2,\cdots i_{k'}} S_{i_1} S_{i_2} \cdots S_{i_{k'}} - h\sum_i S_i$$

- $J_{i1,\ldots ik'}$ of the order k′ is nonzero if the indexes belong to one of the simplexes in the set $\mathcal{F}_{k'}$ of that order; h is the external field;
- Interactions up to the order of the simplicial complex can be formulated;
- [B. Tadic & R. Melnik, Dynamics MDPI (2021)]

Competition of two leading interactions

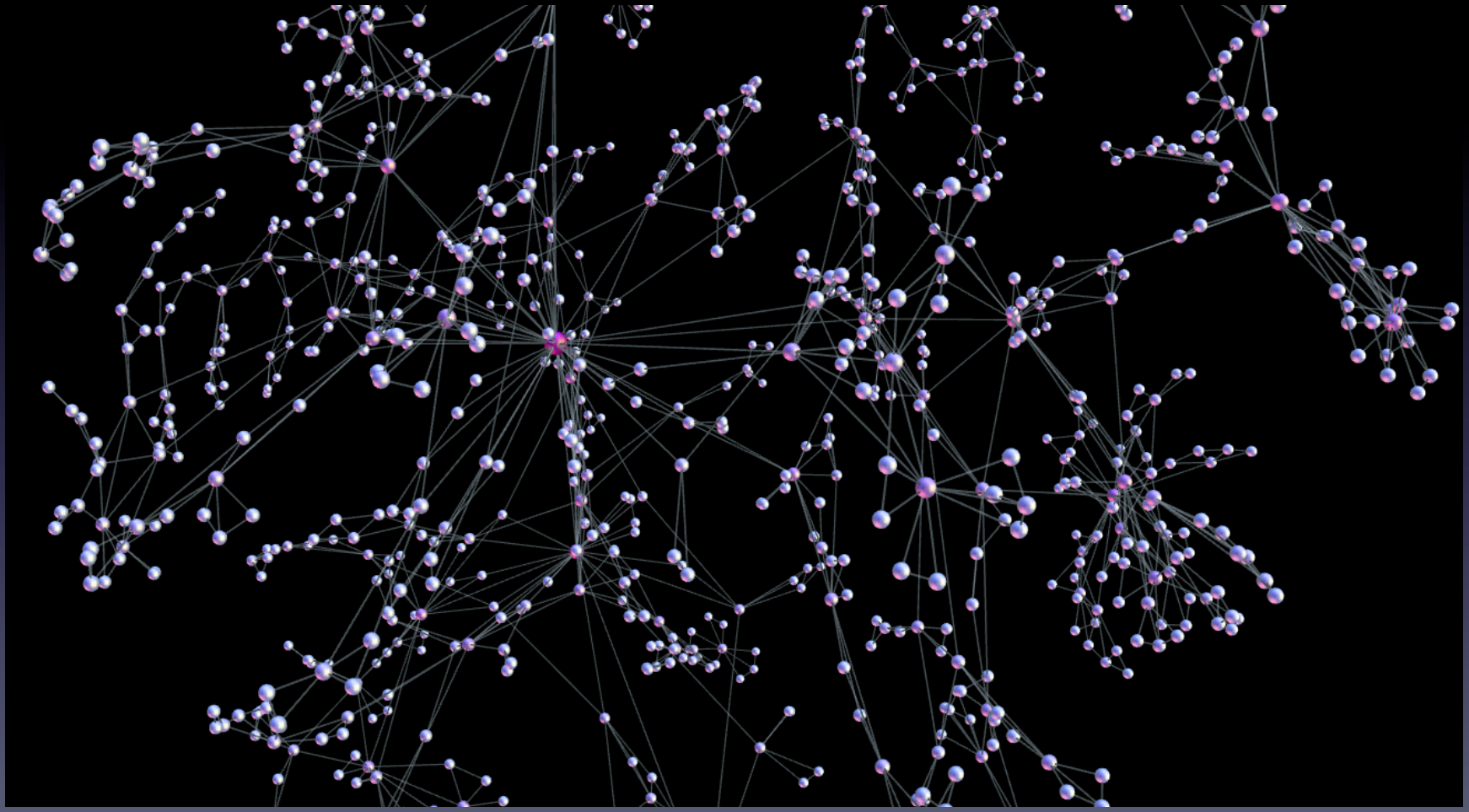
J_{ij} are antiferromagnetic (AF) pairwise interactions embedded on the network's Links (*ij*);
K_{ijk} are 3-spin interaction *on the SC triangles*, combinations of links(*ijk*);

$$\mathcal{H} = (\alpha - 1) \sum_{i,j} J_{ij} S_i S_j - \alpha \sum_{<i,j,k>} K_{ijk} S_i S_j S_k - h_{ext} \sum_i S_i$$

AF pairwise interactions along a triangle cause Spin frustration : orientations of all three spins can not be simultaneously satisfied to minimize the energy. Spin-frustration is a phenomenon occurring in complex materials is utilized for shaping the hysteresis properties;

[B. Tadic & N. Gupte: Europhys. Lett. 132, 60008 (2021)]

Geometric-assembled triangles



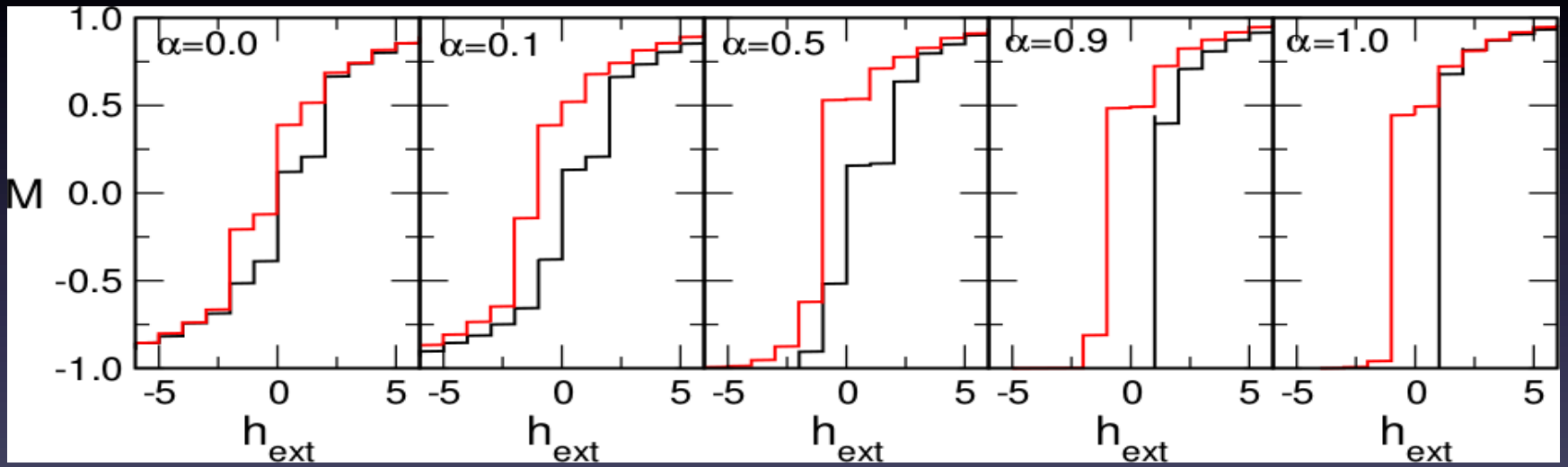
Field-driven Dynamics: Spin reversal on the hysteresis loop

- Start with all spins down & a large negative field $h=-h_{max}$; ($h_{max} = k_{max}$);
- Slowly increase the field h++ (Adiabatically);
- Compute and store *local fields* at each node (sum of neighbor interactions and current value of external field);
- Let the spins rearrange to minimize energy attempting (p) to align with their local fields; (time step t; magnetisation change n_t);
- Continue until no more spins flip (avalanche);
- Increase the field again (a new avalanche);
- Continue until $h=+h_{max}$;
- Reverse the field & repeat the steps until $h=-h_{max}$ (complete the loop);
- [Methods well described in: Tadic, Mijatovic, Janicevic, Spasojevic, Rodgers: Sci. Rep. 9:6340 (2019)]

Impact of the triangle-based interactions: Hysteresis Loop Shape

(1-α) AF pairwise

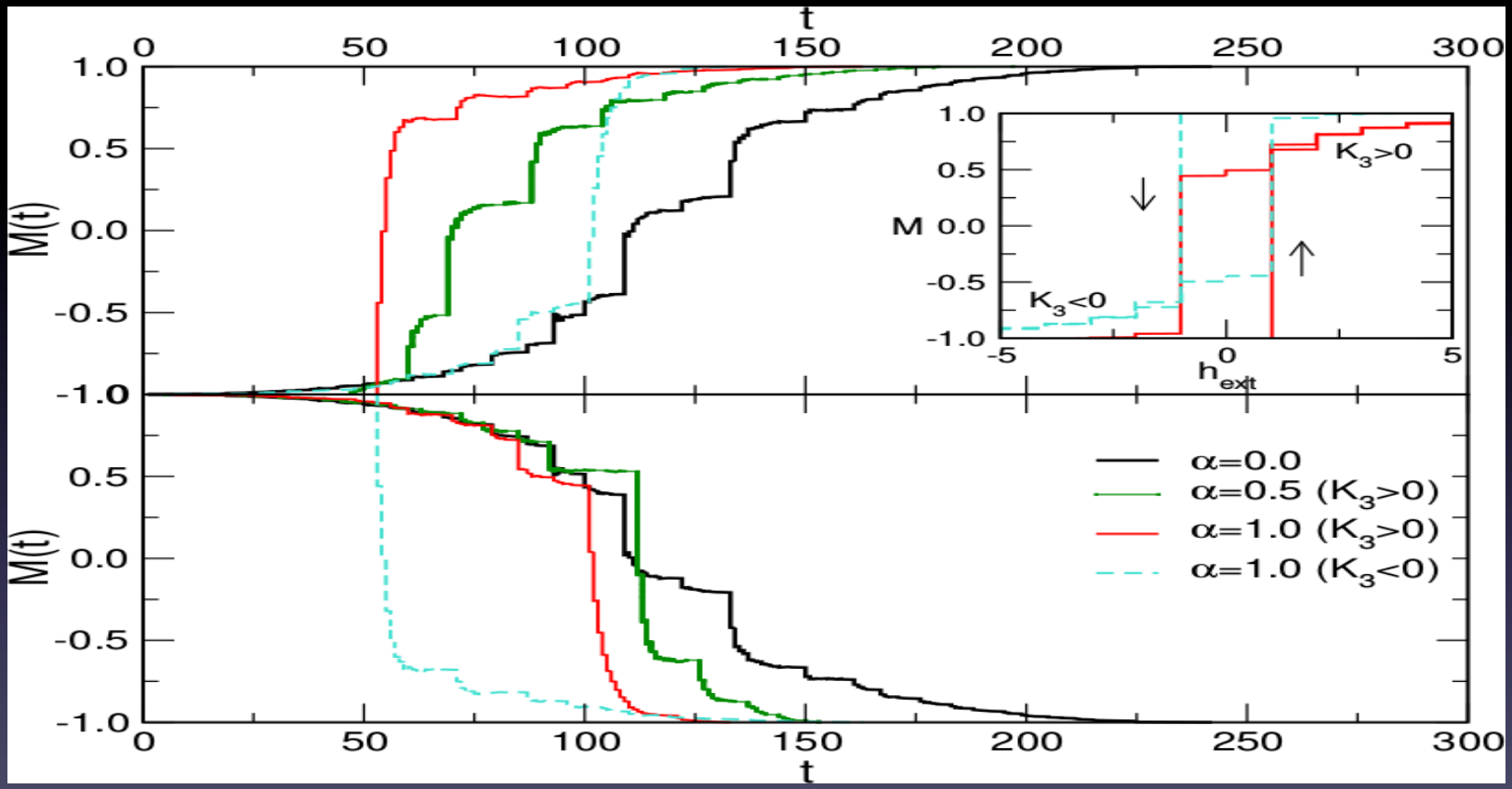
αFM triangle-based



Step-like structure (frustration effects); AF—split loops; Topological disorder (tails);
Increasing the fraction K_3: Asymmetry; Central loop appears, finite M_r; $H_c = 1$ (k_{min}-1);

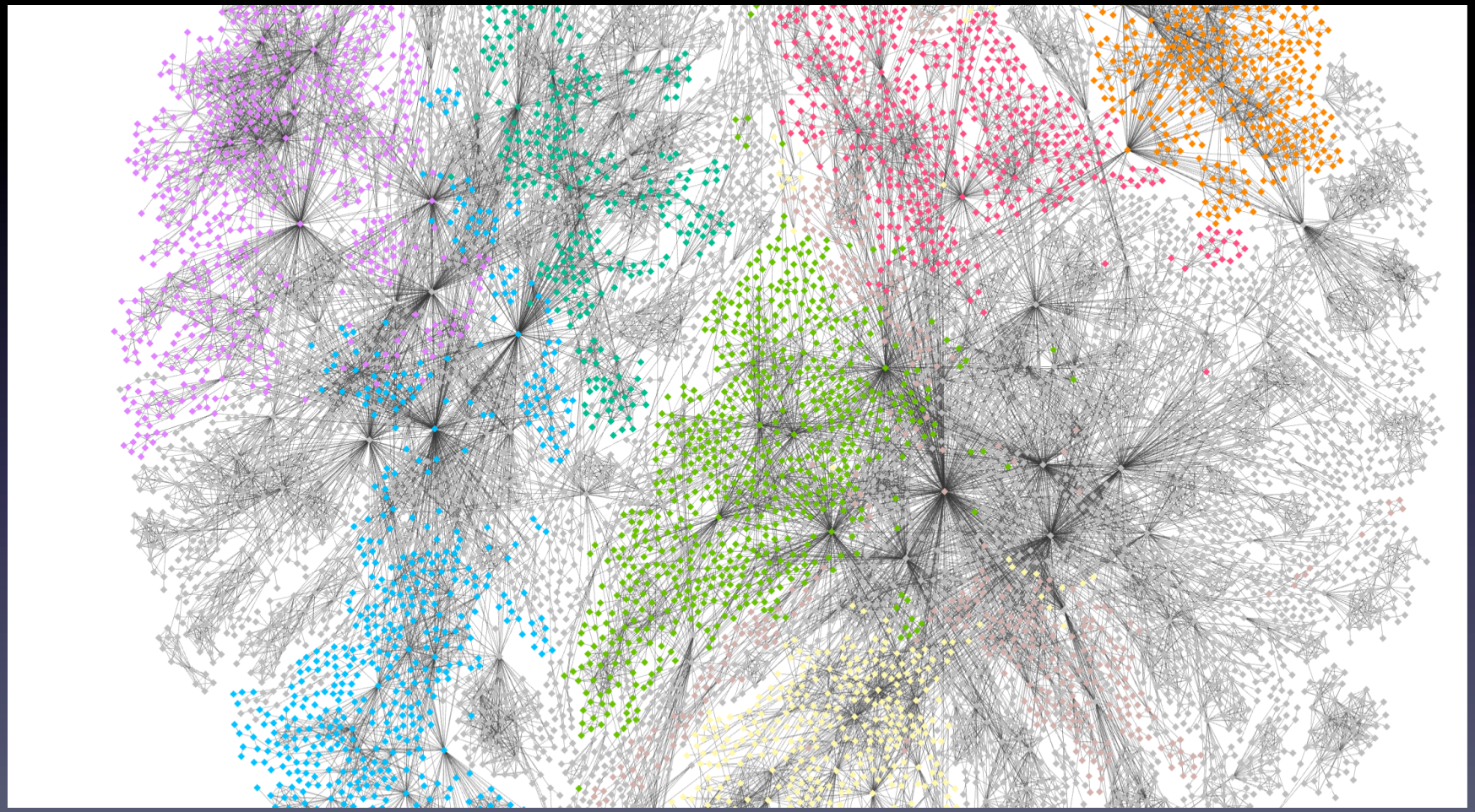
[B. Tadic & N. Gupte: Europhys. Lett. 132:60008 (2021)]

Magnetization bursts between HL plateaus

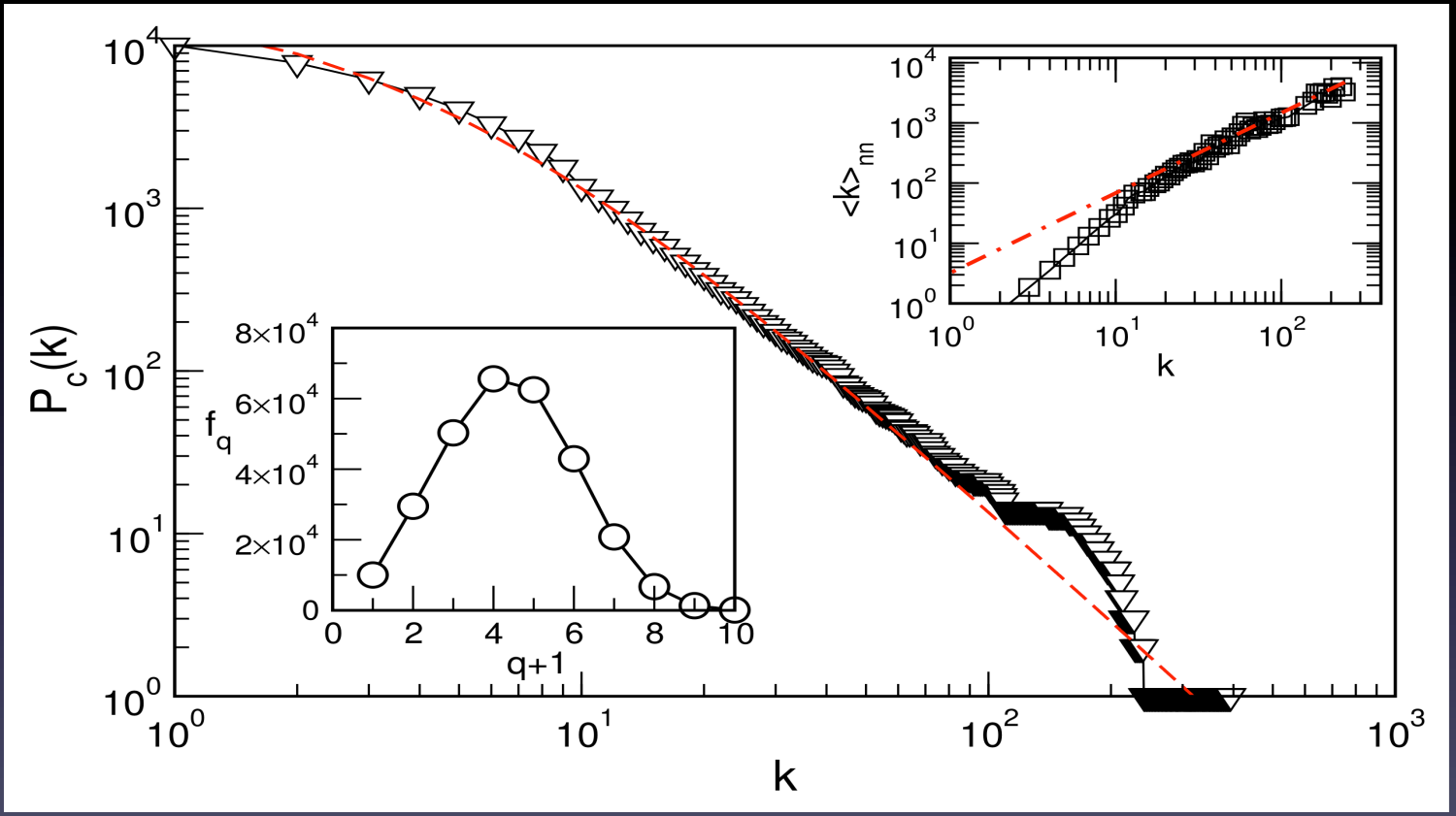


[B. Tadic & N. Gupte: Europhys. Lett. (2021)]

SOC Dynamics: Simplexes of sizes in [2,10]



Network structural properties



Fat-tale degree distribution; assortative correlations of nodes ; hyperbolic; varying number of simplexes along different topology levels q =0,1,2,3,….9

[B. Tadic & R. Melnik, Dynamics MDPI (in review) 2021]

Signatures of Self-Organized Criticality

SOC: In extended interacting systems, the critical state appearing as an attractor of nonlinear dynamics under repeated slow driving [].

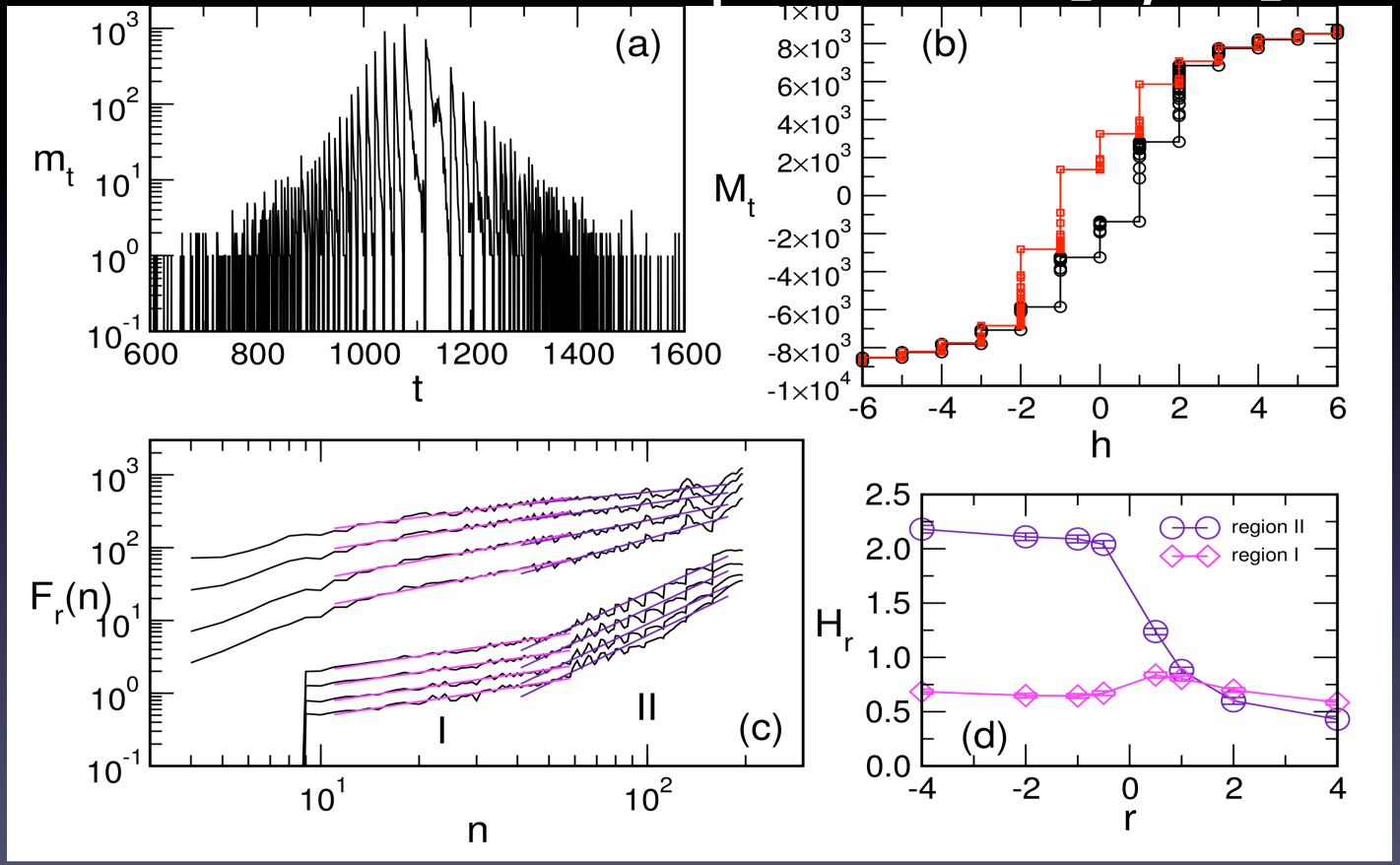
Some features of SOC shown here:

- Long-range temporal correlations: power spectrum $\rightarrow$ $W(f) \sim f^{-\phi}$
- Multifractal fluctuations: generalised Hurst exponent $\rightarrow$ $F_r(n) = \left\{ \frac{1}{2N_s} \sum_{\mu=1}^{2N_s} \left[F^2(\mu, n) \right]^{r/2} \right\}^{1/r} \sim n^{H_r}$
- Avalanches self-similarity: $P_c(s) \sim s^{-(\tau_s - 1)} \mathcal{G}(s, L)$, $P_c(T) \sim s^{-(\tau_T - 1)} \mathcal{D}(T, L)$,... + scaling relations $\gamma_{sT} = \frac{\tau_T - 1}{\tau_s - 1}$; and exhibiting finite-size scaling when L is varied;

These properties can be determined from the output signal (Barkhausen noise), the magnetisation fluctuations time series.

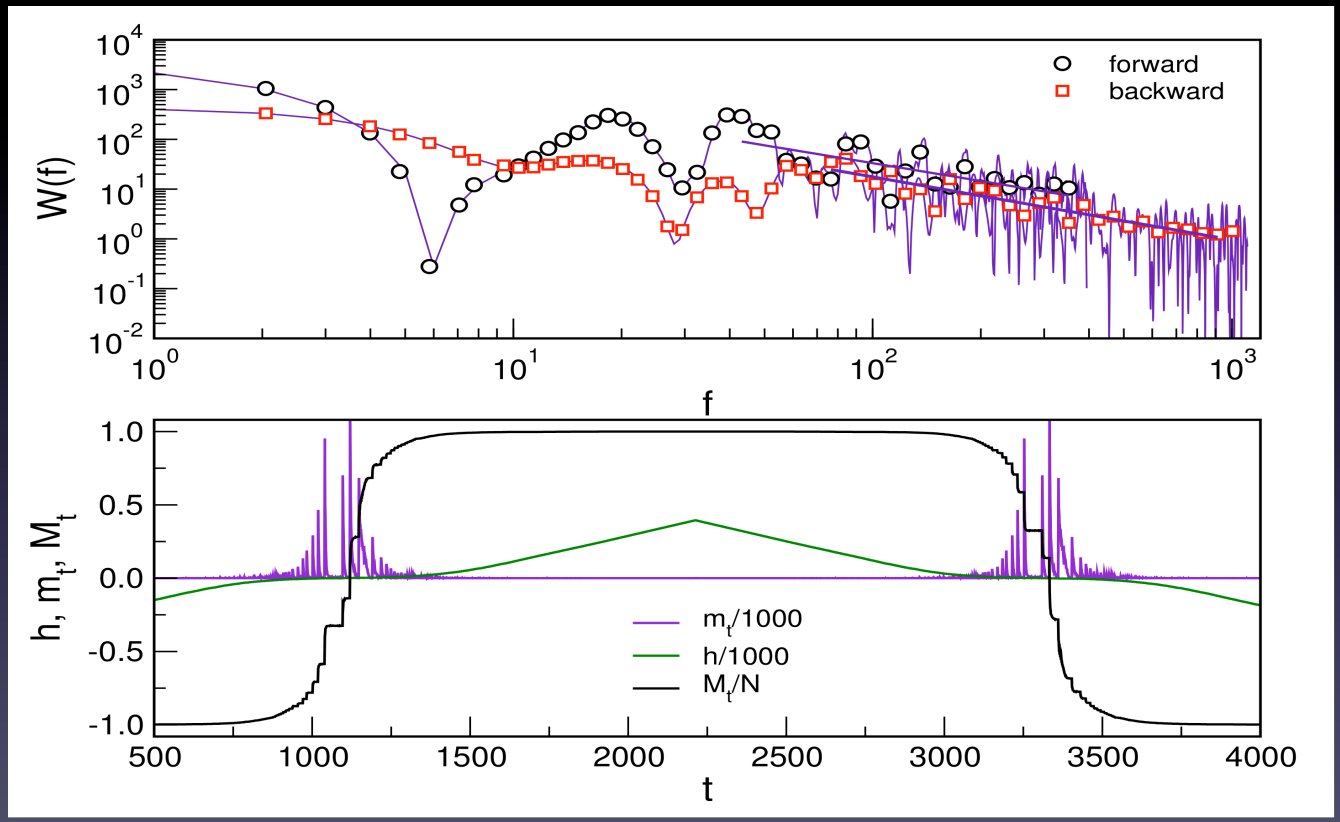
[Marković and Gros, Power laws and self-organized criticality in theory and nature, Phys. Reports 536, 41 (2014)]

Spin frustration (AF pairwise) on a network of different simplexes in [2,10]



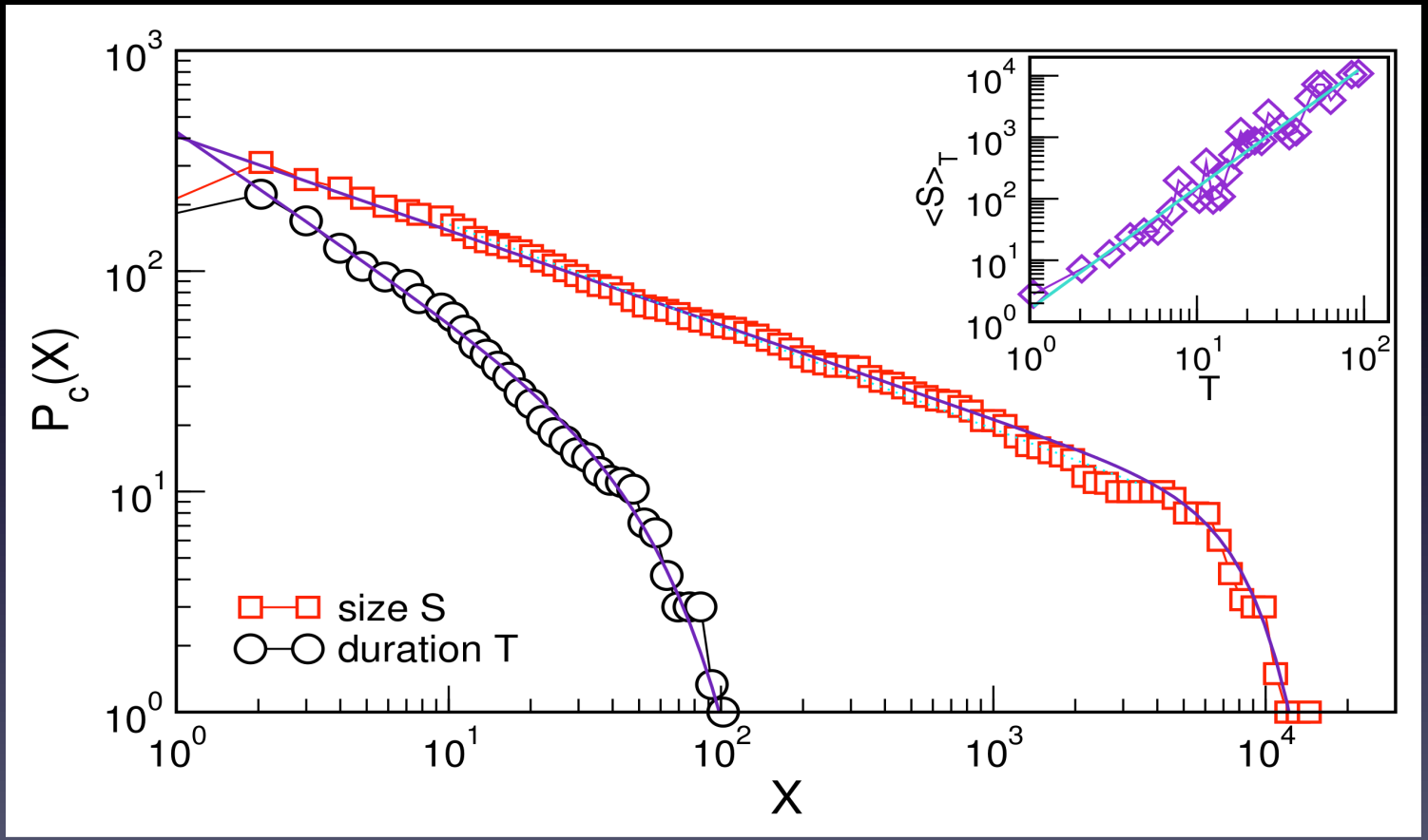
[B. Tadic & N. Melnik: Dynamics MDPI (2021)]

More Features of SOC: Temporal correlations of the magnetisation signal



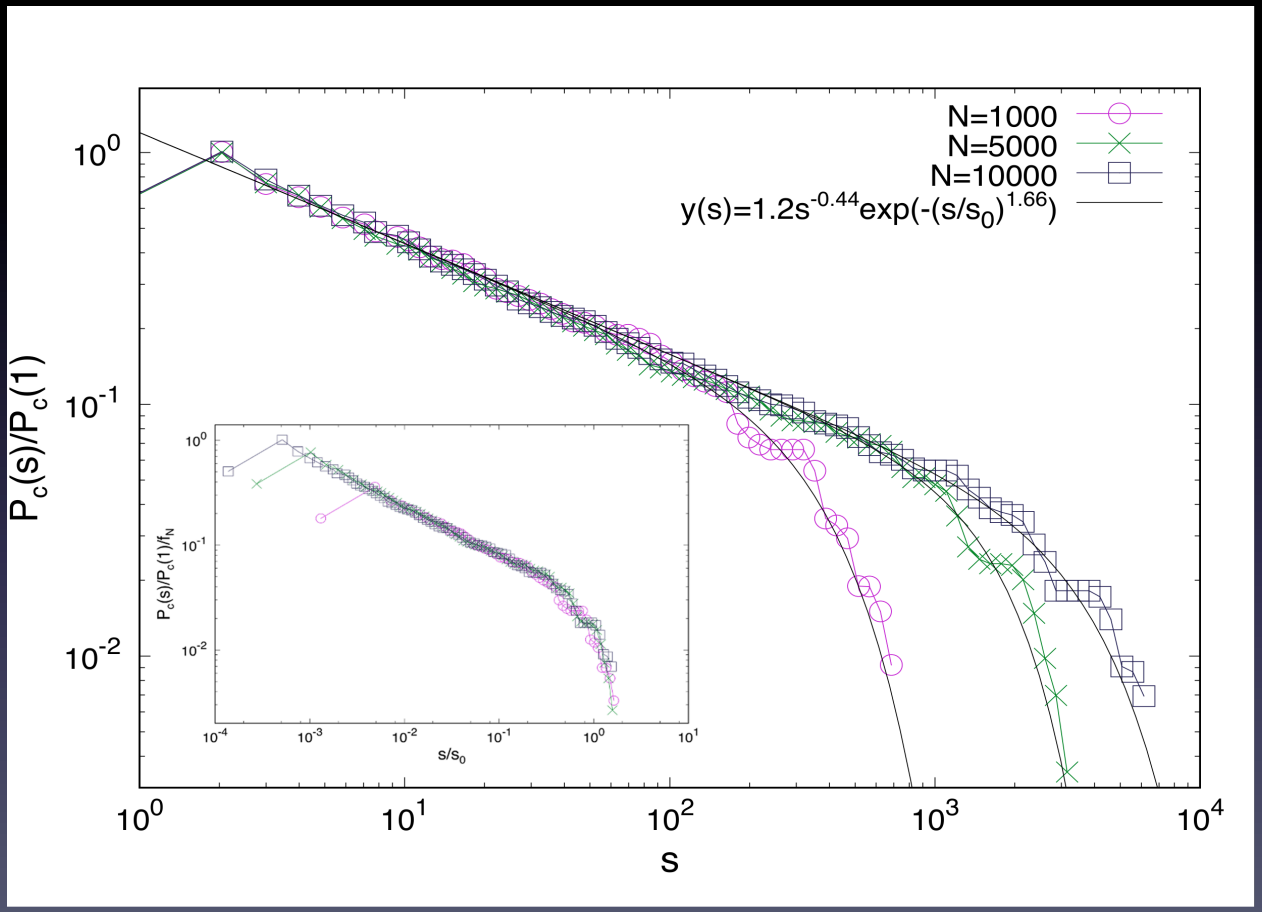
[B. Tadic & N. Melnik: Dynamics MDPI (2021)]

The Magnetization Avalanches



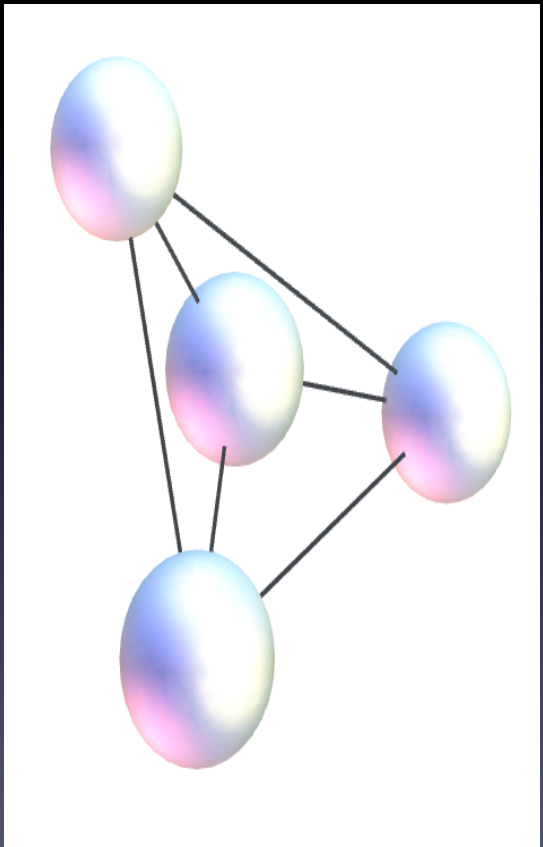
[B. Tadic & N. Melnik: Dynamics MDPI (2021)]

FSS: more signatures of SOC



[B. Tadic & N. Melnik: Dynamics MDPI (2021)]

Summary



- Self-assembled simplexes→to explore the impact of network's structure to the (nonlinear)dynamics;
- Networks' hidden geometries (quantified by Q-analisys) can support a range of higher-order spin interactions;
- AF pairwise couplings on triangle faces → Spin frustration;
- Profound effects on the Hysteresis loop shape and the nature of magnetization fluctuations;
- Long-range temporal correlations, Multi-fractality and Avalanches → Signature of Self-Organized Criticality on HL at the interplay of complex geometry and NL dynamics;

- More... http://www-f1.ijs.si/~tadic/cv_ordered.html#BHN

References

- B. Tadić and R. Melnik, *Self-organized critical dynamics as a key to fundamental features of complexity in physical, biological and social systems*, **Dynamics 1(2), 181-197** (2021)
- B. Tadić and N. Gupte, *Hidden geometty and dynamics of complex systems: spin reversal in complex assemblies with pairwise and triangle-based interactions*, **Europhys. Lett**. 132(6), 60008 (2021)
- B. Tadić, M. Andjelković, M. Šuvakov, G.J. Rodgers, *Magnetization processes in geometrically frustrated spin networks with self-assembled cliques*, **Entropy** 22(3), 336 (2020)
- M. Mitrović Dankulov, B. Tadić, R. Melnik, *Spectral properties of hyperbolic nanonetworks with tunable aggregation of simplexes,* **Phys. Rev. E** 100, 012309 (2019)
- M. Šuvakov, M. Andjelković, B. Tadić, *Hidden geometries in networks arising from cooperative self-assembly*, **Scientific Reports** 8, 1987 (2018)

THANKS !